A Novel Robust Decentralized Adaptive Fuzzy Control for Swarm Formation of Multiagent Systems

Bijan Ranjbar-Sahraei, *Member, IEEE*, Faridoon Shabaninia, *Senior Member, IEEE*, Alireza Nemati, and Sergiu-Dan Stan, *Member, IEEE*

***Abstract*—In this paper, a novel decentralized adaptive control scheme for multiagent formation control is proposed based on an integration of artificial potential functions with robust control techniques. Fully actuated mobile agents with partially unknown models are considered, where an adaptive fuzzy logic system is used to approximate the unknown system dynamics. The robust performance criterion is used to attenuate the adaptive fuzzy approximation error and external disturbances to a prescribed level. The advantages of the proposed controller can be listed as robustness to input nonlinearity, external disturbances, and model uncertainties, and applicability on a large diversity of autonomous systems. A Lyapunov-function-based proof is given of robust stability, which shows the robustness of the controller with respect to disturbances and system uncertainties. Simulation results are demonstrated for a swarm formation problem of a group of six holonomic robots, illustrating the effective attenuation of approximation errors and external disturbances, even in the case of agent failure. Moreover, experimental results confirm the validity of the presented approach and are included to verify the applicability of the scheme for a swarm of six real holonomic robots.**

***Index Terms*—Adaptive fuzzy systems, artificial potential functions, formation control, multiagent systems, robust control.**

I. Introduction

THE EARLY work on robot motion control has considered motion of single robots. However, in recent years, control of a multiagent system consisting a swarm of robots has interested the control communities. One of the main reasons for such interest is to meet the requirement of multiagent systems in industrial and martial applications. Some other reasons are probably the enormous decrement of the cost of single robots or the emerging of new technologies which are capable of making compact robots [1]. Some possible applications of a multiagent system include underwater or outer space exploration, factory transportation, guarding, escorting, and patrolling missions [1], [2].

In general, a multiagent formation problem is defined as the organization of a swarm of agents into a particular shape in a 2-D or 3-D space [2], [3]. In such a problem, a small group of robots can be controlled by a central computer using a centralized approach [4]. However, limitations in computational power and communication bandwidth limit the number of robots. Therefore, decentralized controllers are interested in a large group of robots [5], [6]. Several formation control strategies can be found as potential fields [1], e.g., behavior-based [7], leader-following [8], [9], graph-theoretic [5], and virtual structure approaches [10], [11].

In recent years, some nonlinear control schemes such as sliding-mode control (SMC) have been found very useful in the control of robotic platforms (e.g., [12]). Integration of SMC with potential fields concludes in robust formation control designs for dynamic agents [8]–[11]. For example, Takahashi *et al.* [8] proposed an SMC-based formation control scheme for multiple mobile robots, using the leader-following strategy, in which they defined some performance indexes, so that robots can be controlled according to their ability. Defoort *et al.* [9] also developed a robust coordinated control scheme based on a leader–follower approach to achieve formation maneuvers. They used first- and second-order SMCs to address the formation problem of N mobile robots of unicycle type with two driving wheels. Moreover, Cheaha *et al.* [10] presented a region-based shape controller for a swarm of fully actuated robots, where a linear approximator was used to approximate the unknown dynamic model and an SMC controller integrated with artificial potential functions was used to satisfy a predetermined geometric 2-D shaping.

Recently, H^∞ optimal control techniques have also been found to be an effective solution to treat robust stabilization and tracking problems, in the presence of external disturbances and system uncertainties [13]–[17]. In the traditional H^∞ control, the exact model of the system must be known. However, in order to propose a robust control method, an integration of this robust scheme with fuzzy logic approximators can implement effective controllers for uncertain dynamic models [18].

In this research, a geometric formation is considered as the goal, and a simple artificial potential is defined to guide the agents through this formation. A partially unknown nonlinear dynamic model is adopted to each agent. Therefore, an adaptive fuzzy approximator is combined with H^∞ control technique to propose a novel adaptive fuzzy formation control methodology, with robust characteristics. The main advantage of this control

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B. Ranjbar-Sahraei was with the Department of Power and Control Engineering, Shiraz University, Shiraz 71348-51154, Iran. He is now with the Department of Knowledge Engineering, Maastricht University, 6200 MD Maastricht, The Netherlands (e-mail: b.ranjbar@ieee.org).

F. Shabaninia is with the Department of Power and Control Engineering, School of Electrical and Computer Engineering, Shiraz University, Shiraz 71348-51154, Iran (e-mail: Shabani@shirazu.ac.ir).

A. Nemati is with the School of Mechanical Engineering, Sharif University of Technology, Tehran 11155-8639, Iran (e-mail: nemati@mech.sharif.ir).

S.-D. Stan is with the Department of Mechatronics, Technical University of Cluj-Napoca, 400641 Cluj-Napoca, Romania (e-mail: sergiustan@ieee.org).

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strategy is insensitivity to robot dynamic uncertainties, external disturbances, and input nonlinearities.

The rest of this paper is organized as follows. Section II presents the system description and problem formulation. The design of the proposed controller and stability analysis are discussed in Sections III and IV, respectively. Simulation results are included in Section V, and experimental verification is illustrated in Section VI. Section VII provides the concluding remarks.

II. System Description and Problem Formulation

The major goal in this study is to solve a multiagent formation control problem (i.e., controlling the relative position of the agents to create a desirable formation). One of the effective solutions for this problem is using an electrostatic-like potential function design which guides the agents through continuous smooth paths and avoids agent collisions. Such a potential function design has been discussed in various papers (e.g., [1], [2], and [11]). Therefore, in Section II-A, we will explain a simple potential function design to solve the formation control of a group of N point massless agents, where the kinematics of the ith agent is considered as

$$\dot{z}_i = u_i, \qquad i \in \{1, 2, \ldots, n\} \tag{1}$$

in which $z_i \in \mathbb{R}^N$ is the coordinate matrix (for a robot with n DOF) and $u_i \in \mathbb{R}^n$ denotes the control inputs.

In Section II-B, a general n-DOF dynamic model of real robots is considered to propose more realistic solutions for formation control of multiagent systems. The main feature of this model is that any agent (robot) with n DOF (e.g., autonomous underwater vehicles (AUVs) [19] or unmanned aerial vehicles (UAVs) [20], [21]) can be adopted to this model [22].

A. Formation Control of Massless Agents

To propose a simple control law, an artificial potential function, comprising interagent interactions, environmental effects (e.g., obstacles, goals, etc.), or other exceptional terms, can be designed.

Consider the pairwise potential fields, which are defined between agents as

$$F_{ij} = L_{ij}\left(|z_i - z_j|\right) \qquad \forall i, j \in \{1, 2, \ldots, N\} \tag{2}$$

where L_{ij} is designed to define a proper interagent potential function. It is assumed that each agent senses the resultant potential of all other agents. Therefore, the overall potential function can be proposed to be in the form of

$$F = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} L_{ij}\left(|z_i - z_j|\right) + \sum_{i=1}^{N} Q_i\left(|z_i|\right) \tag{3}$$

where Q_i defines the global potential of each agent.

The potential function in (3) is assumed to be *continuously differentiable* and *positive definite* [10], [11].

To propose a solution for multiagent formation control, the steepest descent direction [1], [10], [11] is chosen for the ith robot as

$$f_i = \nabla_{z_i} F \tag{4}$$

and the control law

$$u_i = -f_i \qquad \forall i \in \{1, 2, \ldots, N\} \tag{5}$$

is proposed.

By substituting (5) in (1), the kinematic model is obtained as

$$\dot{z}_i = -f_i = -\nabla_{z_i} F \qquad \forall i \in \{1, 2, \ldots, N\} \tag{6}$$

which can be rewritten in the matrix form as $\dot{Z} = -\nabla F$, where $Z = [z_1, z_2, \ldots, z_N]$ is the overall generalized coordinate vector.

Remark 1—Agent Collisions: The issue of agent collision is not addressed directly in the proposed method. However, some small modifications on the artificial potential functions (3) can handle this problem. The terms defined in (3) are known as attraction functions. Including interagent repulsion potentials, as discussed in [1], can easily lead to the collision avoidance.

Remark 2—Network Requirements: The computation of the potential function (3) requires that each agent knows its distance to all other agents in the environment. Therefore, the communication topology should be fully connected. In other words, each agent must be either able to compute the distance directly or its neighbors should provide it with their relative distances. Such assumption would be feasible when the number of agents is limited (e.g., the formation control in [3]) or when the agents are equipped with positioning and communication instruments (e.g., the Traxxas Emaxx RC cars equipped with an inertial measurement unit and Global Positioning System used in [2]). However, still, it would be very useful that each agent just uses its distance with limited number of neighbors. To address such kind of limited-communication control strategies, some techniques such as ignoring the distance of far agents or using an estimated configuration vector instead of the exact distance vectors can be used, which are discussed in [23]).

Remark 3—Local Convergence Versus Global Convergence: The gradient-based optimization approaches have inherent local convergence property which might conclude to local solutions. In our proposed gradient-based control method, this property can produce undesired formations. To overcome this issue, gradient-based nonconvex optimization techniques can be used, in which each agent follows the negative gradient of the potential function complemented with a perturbation term. Alternately, designing of convex potential functions can propose another solution. The proper selection of attraction–repulsion forces can conclude to convex potential functions, in which gradient-based methods are guaranteed to find global solutions. For further information, the issue of convexity and nonconvexity of potential functions is studied in [23], and novel designed convex potential functions are proposed in [24].

B. Formation Control of Agents With Dynamic Models

In this section, a general dynamic model is addressed to represent any kind of autonomous n-DOF system. This model has been previously used in some existing works (e.g., [10] and [11]).

Consider a group of N fully autonomous agents. The dynamics of the ith simple agent is nonlinear [22] and can be written in the general form

$$M(z_i)\ddot{z}_i + C(z_i,\dot{z}_i)\dot{z}_i + g(z_i) = u_i + d_i' \tag{7}$$

where $z_i \in \mathbb{R}^n$ is the coordinate matrix (for a robot with n DOF) and $M(z_i) \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix and represents the inertia coefficients. $C(z_i,\dot{z}_i) \in \mathbb{R}^{n\times n}$ is the matrix of centripetal Coriolis damping and rolling resistance forces, $g(z_i) \in \mathbb{R}^n$ is an n-vector of gravitational forces. $u_i \in \mathbb{R}^n$ denotes the control inputs, and $d_i' \in \mathbb{R}^n$ represents the external disturbances.

In most practical control problems of multiagent systems, the inertia matrix $M(z_i)$ is a known constant matrix independent of z_i. Therefore, the following assumption is considered.

Assumption 1: M is the inertia matrix of robots, which is a known and constant matrix.

Using Assumption 1, let us rewrite (7) as

$$M\ddot{z}_i + C(z_i,\dot{z}_i)\dot{z}_i + g(z_i) = u_i + d_i'. \tag{8}$$

It is straightforward to rewrite (8) as

$$\ddot{z}_i = -M^{-1}C(z_i,\dot{z}_i)\dot{z}_i - M^{-1}g(z_i) + M^{-1}u_i + d_i \tag{9}$$

where $d_i = M^{-1}d_i'$.

III. Controller Design Methodology

In this section, a novel formation error based on the integral of formation gradient (4) will be proposed. Then, a robust H^∞ controller will be designed, and a fuzzy logic system (FLS) will be utilized to approximate the unknown parts of dynamic models.

A. Preliminaries

Consider the novel formation error for the ith robot as

$$\underline{e}_i(t) = z_i(t) + \int_0^t f_i(\tau)\,d\tau \tag{10}$$

where $\underline{e}_i \in \mathbb{R}^n$, z_i represents the coordinate vector of the ith robot in (7), and f_i is the gradient of potential function defined in (4). It is straightforward to write the first and second derivatives of (10) as

$$\dot{\underline{e}}_i(t) = \dot{z}_i(t) + f_i \tag{11}$$

$$\ddot{\underline{e}}_i(t) = \ddot{z}_i(t) + \dot{f}_i. \tag{12}$$

Our design goal is to propose a controller so that

$$\ddot{\underline{e}}_i + k_1\dot{\underline{e}}_i + k_2\underline{e}_i = 0 \tag{13}$$

is achieved, where k_1 and k_2 are chosen to make (13) asymptotically stable. Using a feedback linearization method, the controller can be proposed as

$$u_i = M\left(H_i(z_i,\dot{z}_i) - \dot{f}_i - k_1\dot{\underline{e}}_i - k_2\underline{e}_i\right) \tag{14}$$

where

$$H_i(z_i,\dot{z}_i) = M^{-1}C_i(z_i,\dot{z}_i)\dot{z}_i + M^{-1}g(z_i). \tag{15}$$

In order to use this control law, the function $H_i(.)$ [i.e., $C(.)$ and $g(.)$] must be known. However, in practice, these matrices may be unknown for most of real dynamical robots. To overcome this, we make use of an adaptive FLS $\hat{H}_i(.)$ to approximate $H_i(.)$.

Therefore, the overall control law is proposed as

$$u_i = M\left(\hat{H}_i(z_i,\dot{z}_i|\underline{\theta}_i) - \dot{f}_i - k^{\mathrm{T}}\underline{e}_i - \underline{u}_{ai}\right) \tag{16}$$

where $k = [k_1 k_2]^{\mathrm{T}}$ and

$$\hat{H}_i(z_i,\dot{z}_i|\underline{\theta}_i) = \underline{\theta}_i^{\mathrm{T}}\underline{\zeta}_i(z_i,\dot{z}_i) \tag{17}$$

is the fuzzy approximation of H_i, in which $\underline{\theta}_i$ is a vector grouping all adjustable parameters and $\underline{\zeta}_i$ is a set of fuzzy basis functions. Moreover, $\underline{u}_{ai}$ is engaged to attenuate the fuzzy logic approximation error and external disturbances.

B. Description of Adaptive FLSs

An FLS can be employed in adaptive control of nonlinear systems, due to its inherent capabilities of nonlinear function approximation. The basic configuration of an FLS consists of a fuzzifier, a rule base, a fuzzy inference engine, and a defuzzifier. Generally, the rule base can be constructed by the following K fuzzy rules:

$$\begin{aligned} R_i: &\text{IF } x_1 \text{ is } F_1^i \text{ AND } x_2 \text{ is } F_2^i \text{ AND } \ldots \text{ AND } x_n \text{ is } F_n^i \\ &\text{THEN } y_i \text{ is } G^i, i = 1,2,\ldots,K \end{aligned} \tag{18}$$

where $x_1, x_2, \ldots, x_n$ are the FLS inputs and y_i is the ith rule output. F_j^i, $j = 1,2,\ldots,n$, and G^i are fuzzy sets characterized by fuzzy membership functions $\mu_{F_j^i}(x_j)$ and $\mu_{G^i}(y_i)$, respectively, and K is the number of rules in the fuzzy rule base.

Lemma 1: By using the singleton fuzzifier, product inference, and weighted average defuzzifier [25], the output of the FLS can be expressed as

$$y(x) = \frac{\sum_{i=1}^{K} \bar{y}_i \left(\prod_{j=1}^{n} \mu_{F_j^i(x_j)}\right)}{\sum_{i=1}^{K}\prod_{j=1}^{n} \mu_{F_j^i(x_j)}} \tag{19}$$

where $\bar{y}_i$ is the point at which $\mu_{G^i}(y_i) = 1$.

If $\mu_{F_j^i}(x_j)$ denotes the fixed membership functions and $\bar{y}_i$ denotes the adjustable parameters, (19) can be rewritten as

$$y(\underline{x}) = \underline{\theta}^{\mathrm{T}}\underline{\zeta}(\underline{x}) \tag{20}$$

where $\underline{\theta} = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_K]^{\mathrm{T}}$ is a vector grouping all adjustable parameters and $\underline{\zeta}(\underline{x}) = [\zeta_1(\underline{x}), \zeta_2(\underline{x}), \ldots, \zeta_K(\underline{x})]^{\mathrm{T}}$ is a set of fuzzy basis functions defined as

$$\zeta_i(\underline{x}) = \frac{\prod_{j=1}^{n} \mu_{F_j^i(x_j)}}{\sum_{i=1}^{K} \prod_{j=1}^{n} \mu_{F_j^i(x_j)}}. \tag{21}$$

It has been proved in [25] that fuzzy systems in the form of (20) can approximate continuous functions over a compact set to an arbitrary degree of accuracy provided that an enough number of rules are considered.

C. Input Nonlinearity

In practice, due to various phenomena of friction between mechanical parts of a robot, input nonlinearities emerge. Among different input nonlinearities, the dead zone is found very often in robot actuators and is defined as the operating range of input that generates no response in the dynamics of the system output [26]. Based on this definition, the dead-zone effect is more noticeable when the robot is excited with low-amplitude input signals.

Taking into account the dead zone while modeling the multiagent system should result in more realistic models and higher performance controllers. Therefore, various articles have discussed nonlinearity compensation techniques (e.g., [27] and [28]). However, in what follow, it will be shown that the control methodology designed in this paper is inherently robust to dead-zone nonlinearities of the control actuators.

Let us modify the dynamic model (8) as

$$M\ddot{z}_i + C(z_i, \dot{z}_i)\dot{z}_i + g(z_i) = \Phi(u_i) + d_i(t) \tag{22}$$

where

$$\Phi(u_i) = \begin{bmatrix} \phi(u_{i1}) \\ \phi(u_{i2}) \\ \vdots \\ \phi(u_{in}) \end{bmatrix} \tag{23}$$

and $\phi(.) : \mathbb{R} \to \mathbb{R}$ represents the dead-zone function and can be expressed as

$$\phi(u) = \begin{cases} m(u-b), & u \geq b \\ 0, & -b < u < b \\ m(u+b), & u \leq -b \end{cases} \tag{24}$$

where b is the width of the dead zone and m is the slope of lines as in Fig. 1.

The dead-zone parameters b and m are assumed to be bounded, and the bounds of b and m are known as $b \in [b_{\min}, b_{\max}]$ and $m \in [m_{\min}, m_{\max}]$, respectively. Therefore, (24) can be rewritten as

$$\phi(u) = mu + \nu(u) \tag{25}$$

$$\nu(u) = \begin{cases} -mb, & u \geq b \\ -mu, & -b < u < b \\ mb, & u \leq -b. \end{cases} \tag{26}$$

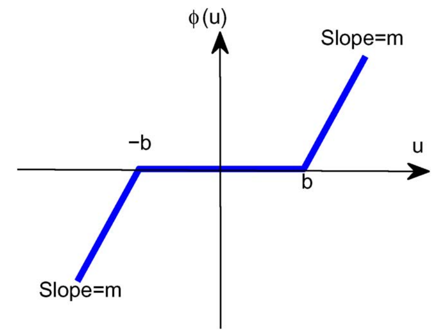

Fig. 1. Dead-zone nonlinearity.

From the aforementioned assumption on bounds of m and b, $\nu(u)$ can be assumed bounded, (i.e., $\nu(u) \leq \lambda$), where λ is the known upper bound that is chosen as $\lambda = mb_{\max}$. By considering $\bar{d}_i(t) = d_i(t) + \Upsilon(u_i)$ and $\bar{u}_i = Mu_i$, where

$$\Upsilon(u_i) = [\nu_1(u_{i1}) \quad \nu_2(u_{i2}) \quad \ldots \quad \nu_n(u_{in})]^{\mathrm{T}}$$
$$M = mI_{n\times n}.$$

then, (22) can be rewritten as

$$M\ddot{z}_i + C(z_i, \dot{z}_i)\dot{z}_i + g(z_i) = \bar{u}_i + \bar{d}_i(t). \tag{27}$$

which is in the same form as (8). Therefore, we have shown that the proposed controller of (16) is also robust to dead-zone input nonlinearities (24).

D. Comparison to the Existing Methods

Although the idea behind the proposed H^∞ controller is close to the SMC-based scheme of Cheaha *et al.* [10], our proposed controller can deal with a much larger class of multiagent systems, with more robustness. In fact, Cheaha *et al.* [10] have ignored nonlinear agent models, external disturbances, and input nonlinearities. They have only considered shape control of a swarm of robots using a linear adaptive approximator, while here, an adaptive nonlinear FLS is used and disturbances integrated with input nonlinearities have been considered.

Moreover, each agent has an adaptive approximator which approximates the unknown dynamics. This approximator just uses the current position and velocity of itself. However, in the work proposed by Cheaha *et al.* [10], the position and velocity of neighbor agents are needed as inputs to the approximator, which makes a high computation complexity in the approximation unit of each agent.

Furthermore, in many existing works on formation control of multiagent systems (e.g., [1], [2], [4], [8], and [9]), limited dynamic or kinematic models as massless agents, unicycles, bicycles, or four-wheeled robots are considered. However, in many real situations, the formation problem of different UAVs, AUVs, and other autonomous systems using a general n-DOF dynamic model is vital, as used in this study.

IV. Stability Analysis

This section presents the stability proof of the proposed adaptive fuzzy controller in (16). A Lyapunov candidate will be proposed, and then, an adaptation law and a robust compensator control input will be derived to satisfy the H^∞ tracking performance.

To derive the adaptive law for adjusting $\underline{\theta}_i$, we first define the optimal parameter vector $\underline{\theta}_i^*$ as

$$\underline{\theta}_i^* = \arg\min_{\underline{\theta}_i \in \Omega_i} \left[\sup \left\| \hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) - H_i(z_i, \dot{z}_i) \right\| \right] \tag{28}$$

where Ω_i is a proper compact set.

The minimum approximation error is defined as

$$\underline{w}'_i = H_i(z_i, \dot{z}_i) - \hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i^*) \tag{29}$$

where $\underline{w}'_i \in L_\infty$ [25].

By choosing the control input as (16), after simple manipulations, the formation error dynamic can be expressed as

$$\ddot{\underline{e}}_i = \left(\hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) - H_i(z_i, \dot{z}_i)\right) - k_1\dot{\underline{e}}_i - k_2\underline{e}_i - \underline{u}_{ai} + d_i. \tag{30}$$

Moreover, by defining $\underline{E}_i = [e_{1i}, \dot{e}_{1i}, e_{2i}, \dot{e}_{2i}, \ldots, e_{ni}, \dot{e}_{ni}]^{\mathrm{T}}$, it is straightforward to write

$$\dot{\underline{E}}_i = A\underline{E}_i + B\underline{u}_{ai} + B\left(H_i(z_i, \dot{z}_i) - \hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i)\right) - Bd_i \tag{31}$$

where

$$A = I_{n\times n} \otimes \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}_{2\times 2} \qquad B = I_{n\times n} \otimes [0 \quad -1]^{\mathrm{T}}$$

where $\otimes$ denotes the Kronecker product.

Based on (17), (28), and (29), the matrix form of formation error in (31) can be rewritten as

$$\dot{\underline{E}}_i = A\underline{E}_i + B\underline{u}_{ai} + B\tilde{\underline{\theta}}_i^{\mathrm{T}} \underline{\zeta}_i(z_i, \dot{z}_i) + B\underline{w}_i \tag{32}$$

where $\tilde{\underline{\theta}}_i = \underline{\theta}_i - \underline{\theta}_i^*$ and $\underline{w}_i = \underline{w}'_i - d_i$.

In the following theorem, it will be proved that the proposed control law (16) guarantees the stability and robustness of the formation problem.

Theorem 1: Consider a group of N fully autonomous agents with the dynamics represented in (9) and with the control law in (16). The robust compensator of the ith robot $\underline{u}_{ai}$ and the fuzzy adaptation law are chosen as

$$\underline{u}_{ai} = -\frac{1}{r}B^{\mathrm{T}}P\underline{E}_i \tag{33}$$

$$\dot{\underline{\theta}}_i = -\gamma\underline{\zeta}(z_i, \dot{z}_i)\underline{E}_i^{\mathrm{T}}PB \tag{34}$$

where r and γ are positive constants and P is the positive semidefinite solution of the following Riccati-like equation:

$$PA + A^{\mathrm{T}}P + Q - \frac{2}{r}PBB^{\mathrm{T}}P + \frac{1}{\rho^2}PBB^{\mathrm{T}}P = 0 \tag{35}$$

where Q is a positive semidefinite matrix and $2\rho^2 \geq r$.

Therefore, the H^∞ tracking performance

$$\sum_{i=1}^N \left[\int_0^{\mathrm{T}} \underline{E}_i^{\mathrm{T}} Q \underline{E}_i dt\right] \leq \sum_{i=1}^N \left[\underline{E}_i(0)^{\mathrm{T}} P\underline{E}_i(0) + \frac{1}{\gamma}tr\left(\tilde{\underline{\theta}}_i^{\mathrm{T}}(0)\tilde{\underline{\theta}}_i(0)\right)\right] + \rho^2 \sum_{i=1}^N \left[\int_0^{\mathrm{T}} \underline{w}_i^{\mathrm{T}}\underline{w}_i dt\right] \tag{36}$$

can be achieved for a prescribed attenuation level ρ, and all the variables of a closed-loop system remain bounded.

Proof: In order to derive the adaptive law for adjusting $\underline{\theta}_i$, the Lyapunov candidate is chosen as

$$V = \sum_{i=1}^N \left[\frac{1}{2}\underline{E}_i^{\mathrm{T}}P\underline{E}_i + \frac{1}{2\gamma}tr\left(\tilde{\underline{\theta}}_i^{\mathrm{T}}\tilde{\underline{\theta}}_i\right)\right]. \tag{37}$$

Using (32), the time derivative of V is

$$\begin{aligned}\dot{V} = \frac{1}{2}\sum_{i=1}^N \Big[&\underline{E}_i^{\mathrm{T}}A^{\mathrm{T}}P\underline{E}_i + \underline{u}_{ai}^{\mathrm{T}}B^{\mathrm{T}}P\underline{E}_i + \underline{\zeta}_i^{\mathrm{T}}(z_i, \dot{z}_i)\tilde{\underline{\theta}}_i B^{\mathrm{T}}P\underline{E}_i \\ &+ \underline{w}_i^{\mathrm{T}}B^{\mathrm{T}}P\underline{E}_i + \underline{E}_i^{\mathrm{T}}PA\underline{E}_i + \underline{E}_i^{\mathrm{T}}PB\underline{u}_{ai} \\ &+ \underline{E}_i^{\mathrm{T}}PB\tilde{\underline{\theta}}_i^{\mathrm{T}}\underline{\zeta}_i(z_i, \dot{z}_i) + \underline{E}_i^{\mathrm{T}}PB\underline{w}_i\Big] \\ + \frac{1}{2}\sum_{i=1}^N &\left[\frac{1}{\gamma}tr\left(\dot{\tilde{\underline{\theta}}}_i^{\mathrm{T}}\tilde{\underline{\theta}}_i\right)\frac{1}{\gamma}tr\left(\tilde{\underline{\theta}}_i^{\mathrm{T}}\dot{\tilde{\underline{\theta}}}_i\right)\right].\end{aligned} \tag{38}$$

Substituting (33) in (38) and using the fact that $\dot{\tilde{\underline{\theta}}}_i = \dot{\underline{\theta}}_i$, we get

$$\begin{aligned}\dot{V} = &\frac{1}{2}\sum_{i=1}^N \left[\underline{E}_i^{\mathrm{T}}\left(A^{\mathrm{T}}P + PA - \frac{2}{r}\underline{P}BB^{\mathrm{T}}P\right)\underline{E}_i\right] \\ &+ \frac{1}{2}\sum_{i=1}^N \left[tr\left(\tilde{\underline{\theta}}_i^{\mathrm{T}}\left(\underline{\zeta}_i(z_i, \dot{z}_i)\underline{E}_i^{\mathrm{T}}PB + \frac{1}{t\gamma}\dot{\underline{\theta}}_i\right)\right)\right] \\ &+ \frac{1}{2}\sum_{i=1}^N \left[\underline{w}_i^{\mathrm{T}}B^{\mathrm{T}}P\underline{E}_i + \underline{E}_i^{\mathrm{T}}PB\underline{w}_i\right].\end{aligned} \tag{39}$$

Using the adaptation law (34) and the Riccati-like equation (35), (39) becomes

$$\begin{aligned}\dot{V} = \frac{1}{2}\sum_{i=1}^N \Bigg[&-\underline{E}_i^{\mathrm{T}}Q\underline{E}_i \\ &- \left(\frac{1}{\rho}B^{\mathrm{T}}P\underline{E}_i - \rho\underline{w}_i\right)^{\mathrm{T}}\left(\frac{1}{\rho}B^{\mathrm{T}}P\underline{E}_i - \rho\underline{w}_i\right)\Bigg] \\ &+ \frac{1}{2}\sum_{i=1}^N \left[\rho^2\underline{w}_i^{\mathrm{T}}\underline{w}_i\right] \\ \leq \frac{1}{2}\sum_{i=1}^N &\left[-\underline{E}_i^{\mathrm{T}}Q\underline{E}_i + \rho^2\underline{w}_i^{\mathrm{T}}\underline{w}_i\right].\end{aligned} \tag{40}$$

Integrating the above inequality from $t = 0$ to T yields

$$V(T)-V(0)\leq\frac{1}{2}\sum_{i=1}^{N}\left[-\int_0^T \underline{E}_i^{\mathrm{T}}Q\underline{E}_i dt+\rho^2\int_0^T \underline{w}_i^{\mathrm{T}}\underline{w}_i dt\right]. \quad (41)$$

Using the fact that $V(T) \geq 0$ and from (37), the inequality (36) can be computed from (41). Therefore, the H^∞ criterion is achieved, and the proof is completed. ■

The proposed control methodology for the ith robot can be summarized in Algorithm 1.

Algorithm 1 Formation Control Algorithm for the ith Agent
Initialize θ_i
loop
  Approximate the distance with other robots
  Compute the potential function according to (3)
  Compute the steepest decent direction according to (4)
  Compute the formation error according to (10)
  Update θ_i according to (34)
  Make the approximation $\hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) = \underline{\theta}_i^{\mathrm{T}}\underline{\zeta}_i(z_i, \dot{z}_i)$
  Compute the robust controller u_{ai} according to (33)
  Apply controller $u_i = M(\hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) - \dot{f}_i - k^{\mathrm{T}}\underline{e}_i - \underline{u}_{ai})$
end loop

Remark 4—Communication Delay: Although communication delay is an important issue to the multiagent systems, for simplicity, we did not consider this case in the aforementioned stability proof. On one side, as our formation control scheme is fully decentralized, it does not need direct interagent communications. On the other side, the time constant of mechanical actuators is much larger than the sensor time delays. Therefore, the precise stability proof under time-delay conditions is beyond the main objective of this paper. For more information, refer to [29].

V. Simulation Results

This section presents four simulation examples to illustrate the effectiveness of the proposed control scheme. The first example presents the hexagonal formation of six partially unknown agents. This example shows the system stability under the proposed novel controller. In the second example, a white Gaussian noise is applied to all measured data; then, one of the agents is forced to be stationary, but still, the formation maintains its performance. In the third example, one of the agents is chosen as the leader with a constant velocity, and it is shown that the proposed controller is able to form a hexagon which tracks the leader. Then, in the fourth example, a comparison between the proposed method and an existing SMC-based method will be presented, in which the effectiveness of our control scheme will be shown.

TABLE I
Parameter Specifications of Hexagonal Formation

	|i-j|=1	|i-j|=2	|i-j|=3	|i-j|=4	|i-j|=5
d_{ij}	1.0	1.7	2.0	1.7	1.0

TABLE II
Agent Initial Positions

Agent No:	*1*	*2*	*3*	*4*	*5*	*6*
x_0	-2.5	+2.0	-1.0	+1.0	+2.0	+2.5
y_0	+1.0	-2.5	+1.0	-1.0	+2.5	-1.0

The unique formation problem used in all four simulation examples is a 2-D hexagon with unit radius defined by

$$F = \sum_{i=1}^{5}\sum_{j=i+1}^{6}\left(|z_i - z_j|^2 - d_{ij}\right)^2 \quad (42)$$

where d_{ij} is specified in Table I.

In addition, six random points in the 2-D space are chosen to be the initial positions of the six agents (Table II).

Based on the general model representation of robots in (9), the nonlinear dynamics of the ith robot is considered as

$$\begin{bmatrix}\ddot{x}_i\\ \ddot{y}_i\end{bmatrix} = -\begin{bmatrix}0.25 & 0\\ 0 & 0.25\end{bmatrix}\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\end{bmatrix} - \begin{bmatrix}0.33 & 0\\ 0 & 0.33\end{bmatrix}\begin{bmatrix}\mathrm{sgn}(\dot{x}_i)\\ \mathrm{sgn}(\dot{y}_i)\end{bmatrix} + \begin{bmatrix}1.66 & 0\\ 0 & 1.66\end{bmatrix}u_i. \quad (43)$$

A. Example I. Six Agents With Partially Unknown Dynamics

Consider a group of six agents with the dynamic models as (43). The formation potential and the formation error are chosen as (42) and (10), respectively, where $k_1 = 15$ and $k_2 = 4$. To design the control law, the dynamic model of agents is assumed to be partially unknown [i.e., $C(.)$ and $g(.)$ in (7) are unknown].

Therefore, six fuzzy logic approximators are designed to approximate the unknown dynamics, where each agent approximator just needs the current position and velocity of itself. Gaussian membership functions are defined as follows:

$$\mu_{F_1^1}(x) = \frac{1}{1+\exp\left(3(x+0.5)\right)}$$
$$\mu_{F_1^2}(y) = \frac{1}{1+\exp\left(3(y+0.5)\right)}$$
$$\mu_{F_1^3}(x) = \frac{1}{1+\exp\left(-3(x-0.5)\right)}$$
$$\mu_{F_1^4}(y) = \frac{1}{1+\exp\left(-3(y-0.5)\right)}$$
$$\mu_{F_2^1}(\dot{x}) = \frac{1}{1+\exp\left(30(\dot{x}+0.15)\right)}$$
$$\mu_{F_2^4}(\dot{y}) = \frac{1}{1+\exp\left(30(\dot{y}+0.15)\right)}$$
$$\mu_{F_2^2}(\dot{x}) = \exp(-30\times\dot{x}^2)$$
$$\mu_{F_2^5}(\dot{y}) = \exp(-30\times\dot{y}^2)$$
$$\mu_{F_2^3}(\dot{x}) = \frac{1}{1+\exp\left(-30(\dot{x}-0.15)\right)}$$
$$\mu_{F_2^6}(\dot{y}) = \frac{1}{1+\exp\left(-30(\dot{y}-0.15)\right)}.$$

Using the aforementioned 10 membership functions, 13 fuzzy rules are designed as

$$R_l : \text{IF } x \text{ is } F_1^i \text{ AND } y \text{ is } F_1^j \text{ THEN } y\prime \text{ is } G_1^{ij},$$
$$i = 1, 2; \quad j = 3, 4$$
$$R_l : \text{IF } \dot{x} \text{ is } |F_2^i \text{ AND } \dot{y} \text{ is } F_2^j \text{ THEN } y\prime \text{ is } G_2^{ij},$$
$$i = 1, 2, 3; \quad j = 4, 5, 6$$

where x and y are the position coordinates of an individual robot and $y\prime$ is the output of each rule.

The output of the fuzzy system is achieved by choosing singleton fuzzification, center average defuzzification, Mamdani implication in the rule base, and product inference engine [25] as

$$\hat{H}(x, y, \dot{x}, \dot{y}|\underline{\theta}) = \left[\underline{\theta}_1^{\mathrm{T}} \underline{\zeta}(x, y, \dot{x}, \dot{y}) \underline{\theta}_2^{\mathrm{T}} \underline{\zeta}(x, y, \dot{x}, \dot{y})\right]^{\mathrm{T}}$$

where

$$\underline{\theta}_1 = [\theta_{1_1}, \theta_{1_2}, \ldots, \theta_{1_{13}}]^{\mathrm{T}}$$
$$\underline{\theta}_2 = [\theta_{2_1}, \theta_{2_2}, \ldots, \theta_{2_{13}}]^{\mathrm{T}}$$

are adjustable parameters and

$$\underline{\zeta}(x, y, \dot{x}, \dot{y}) = \frac{\left[\mu_{F_1^1} \cdot \mu_{F_1^3} \cdots \mu_{F_1^2} \cdot \mu_{F_1^4} \; \mu_{F_2^1} \cdot \mu_{F_2^4} \cdots \mu_{F_2^3} \cdot \mu_{F_2^6}\right]^{\mathrm{T}}}{D}.$$

All $\underline{\theta}$'s are initialized from zero vectors, and the learning rate in (34) is set to $\gamma = 15$.

The proposed adaptive fuzzy H^∞ technique is applied to the simulated multiagent system with agent initial positions as shown in Table II. The motion trajectories in the first 30 s are shown in Fig. 2(a), and the formation potential (42) is shown to be stabilized in Fig. 2(b). In addition, in order to illustrate the fuzzy approximator performance, identification errors $\|\hat{H}_i(z, \dot{z}_i|\theta_i) - H_i(z_i, \dot{z}_i)\|$ of the first and second robots are shown in Fig. 3(a) and (b), respectively (the rest of the robots have similar identification error figures).

B. Example II. Formation Problem in the Presence of Measurement Noise and Agent Failure

In this example, the robustness of the proposed controller in the presence of measurement noise and agent failure will be shown. The proposed potential function (3) and gradient-based method proposed in Section II have the potential to obtain the exact formation even in the case of one agent failure. Therefore, Agent #3 ($x_3(0) = -1, \; y_3(0) = +1$) is forced to be stationary with zero velocity. In addition, a white Gaussian noise with $SNR = 20$ db is applied to all measured data. Motion trajectory and formation potential (42) of the first 90 s of the simulation are shown in Fig. 4(a) and (b), respectively.

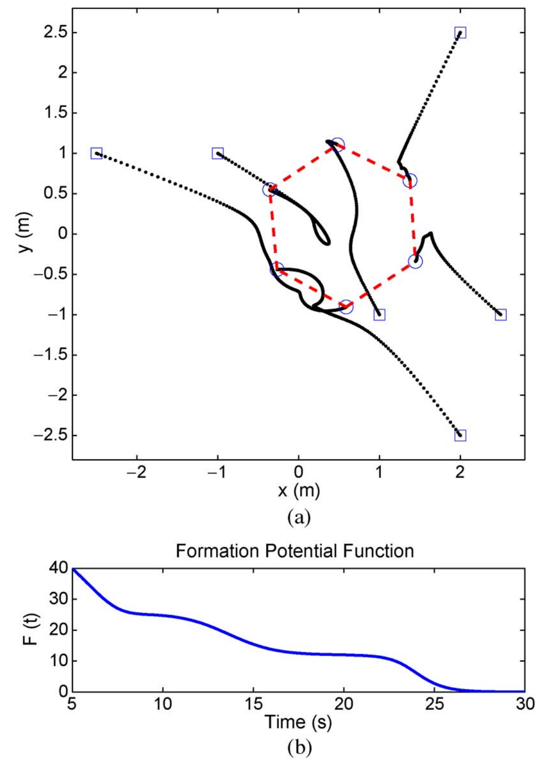

Fig. 2. Hexagonal formation of six agents with partially unknown dynamics. (a) Formation trajectory. (b) Formation potential.

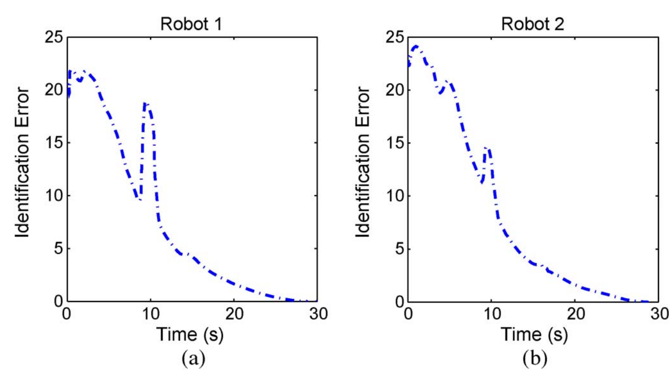

Fig. 3. Identification error during formation control. (a) Identification error of the first robot. (b) Identification error of the second robot.

C. Example III. Formation Problem While Tracking the Leader

The structure of potential function explained in (3) suggests to exempt one agent from the control law designed in (16) and let it move freely as the leader. Therefore, to run a more general simulation than the previous example where one agent was stationary, here, one of the agents is chosen as the leader and moves with a constant speed to a predefined direction. Then, it is anticipated that, after some transient formation, the agent position achieves the hexagon form in (42).

All the problem parameters and controller designs are the same as those in the previous examples. Agent #4 ($x_4(0) = +1$, $y_4(0) = -1$) is chosen as the leader, with constant velocity as

$$\begin{cases} \dot{x}_4 = +0.030 \\ \dot{y}_4 = -0.005. \end{cases}$$

The motion trajectory and formation potential (42) are shown in Fig. 5(a) and (b), respectively.

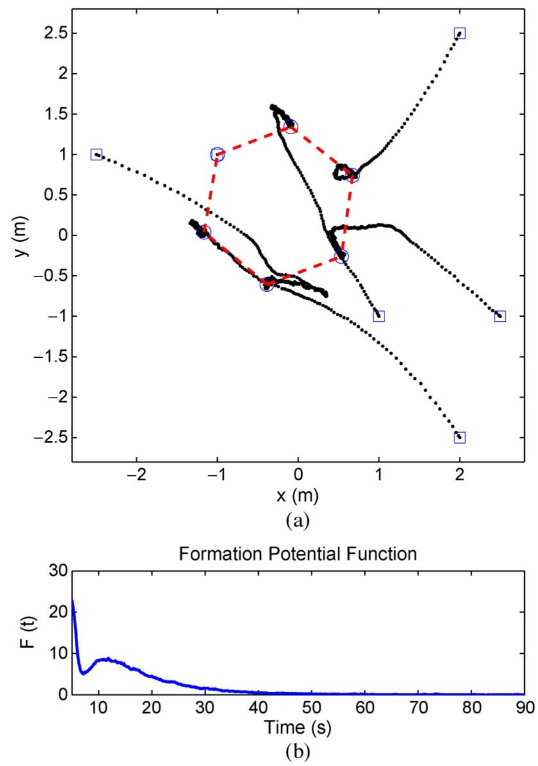

Fig. 4. Hexagonal formation in the presence of 20-db noise and one agent failure. (a) Formation trajectory. (b) Formation potential.

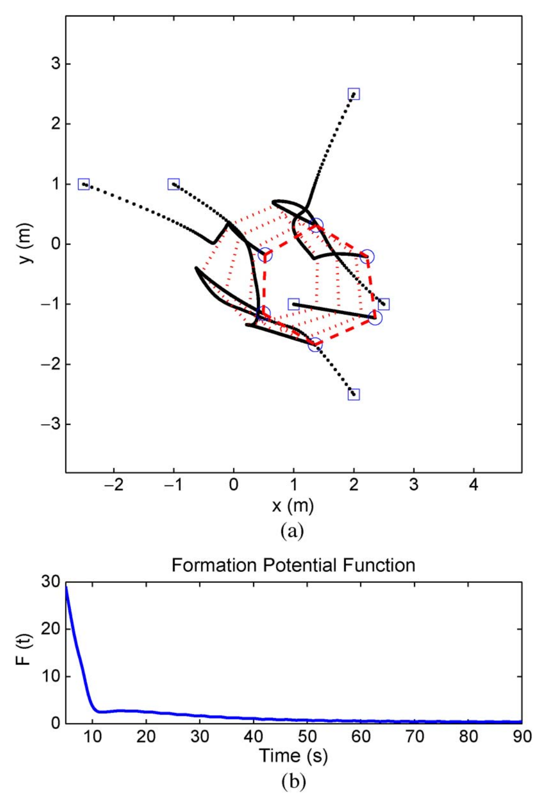

Fig. 5. Moving hexagonal formation while tracking the leader. (a) Formation trajectory. (b) Formation potential.

Simulation results illustrate that, by using the same control law as (16), even the moving formation is achieved. It can be seen that the formation is achieved in about 60 s; however, this numerical simulation contains negligible steady-state error.

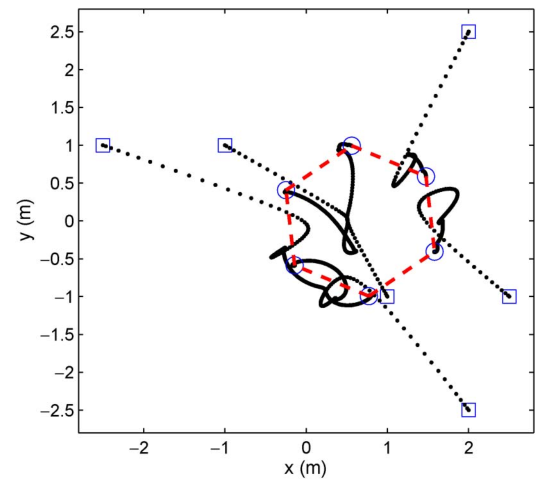

Fig. 6. Hexagonal formation trajectory for SMC-based control scheme [10].

D. Example IV. Comparison of the Proposed Controller With an Existing Method

The proposed control scheme has two key features. First, it is based on H^∞ control technique, which guarantees its robustness to uncertainties, and second, it uses adaptive FLSs to approximate the unknown dynamics. Recently, some researchers have suggested SMC-based controllers to solve the similar problem (e.g., [10] and [11]). Cheaha *et al.* proposed one of these SMC-based methods [10]. Therefore, in this section, the same potential function as in (42) and the same dynamic model as in (43) are chosen for the controller in [10]. However, the shape control potential in [10] is exchanged for formation control potential (3). To design the SMC input [10], a sliding surface is considered as

$$s_i = \dot{z}_i + f_i \tag{44}$$

where f_i is defined in (4) and the control law is implemented using

$$u_i = -K_{si} - K_p f_i + Y_i \hat{\theta}_i \tag{45}$$

in which Y_i is a known regressor matrix composed of agent position and velocity. Then, the adaption law is considered as

$$\dot{\hat{\theta}}_i = -L_i Y_i^{\mathrm{T}} s_i \tag{46}$$

where the constant matrices are chosen as

$$K_{si} = \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} \qquad L_i = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

$$K_p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

to implement numerical simulations. The main strategy in [10] is to force the agents to satisfy the sliding surface in (44).

The hexagonal formation made by this method is shown in Fig. 6. The same problem was previously solved by our proposed method in Section V-A [Fig. 2(a)].

Fig. 6 shows the agent motion trajectory based on the method discussed in [10]. It can be seen that agents move through

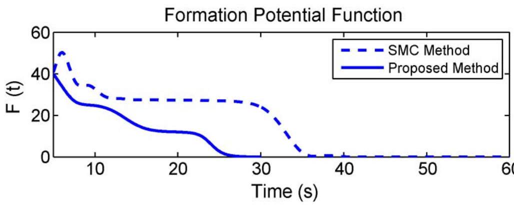

Fig. 7. Formation potential function decrement comparison between SMC-based control scheme [10] and the proposed control method.

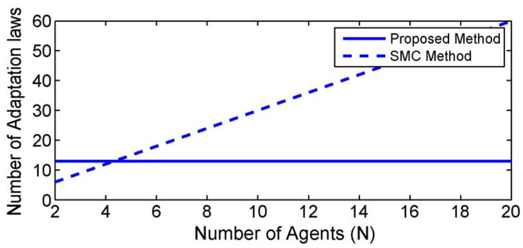

Fig. 8. Computational complexity comparison of SMC-based method [10] with the proposed control method.

nonsmooth and long paths to form the hexagon. Such undesired transient motions are caused by approximation errors related to large number of input parameters and inability to approximate nonlinear functions [e.g., the $\mathrm{sgn}(.)$ in (43)].

To compare the SMC-based method with the proposed H^∞ control scheme, Fig. 7 presents a comparison which shows the trend of potential decrement in the first 60 s of the numerical simulation.

To compare the computational complexities of the two methods, consider N agents, each one with 2-DOF. The proposed method uses the position and velocity of each agent for itself. As described in first simulation example (Section V-A), each agent needs 13 adaptation rules. As it can be seen, this number is independent of N. However, in the control method of Cheaha *et al.*, a collection of $4N$ adaptation laws are needed. Fig. 8 shows the computational complexity comparison for different swarm populations (N). However, it should be mentioned that, when the main goal is to control the formation of a group of n-DOF robots $(n \geq 3)$, then the linear-approximation-based methods, such as the method of Cheaha *et al.*, can have less computational complexity compared to the fuzzy-approximation-based methods.

VI. Experimental Verification

In this section, to verify the efficiency of the proposed method, the control algorithm is implemented on a swarm of six holonomic Palm Pilot Robot Kit (PPRK) robots [Fig. 9(a)]. Each PPRK mass is 450 g, and its radius is equal to 11 cm. PPRKs cannot reach to an acceleration more than 400 cm/s^2 and a velocity more than 25 cm/s.

The robotic swarm moves on a 250 cm $\times$ 320 cm black rigid surface. A digital camera is installed at a height of 3.5 m from the surface, and image-processing software uses $15f/s$ of an image with 600 $\times$ 800 pixels to exactly denote the place of each robot. The required raw data consisting of local distances between robots are sent to each of them, and the whole communication is set up by wireless frequency-shift-keying modules. Each robot uses the received data to choose the proper velocity and direction of motion. The whole arrangement is very similar to the experimental test bed in [9].

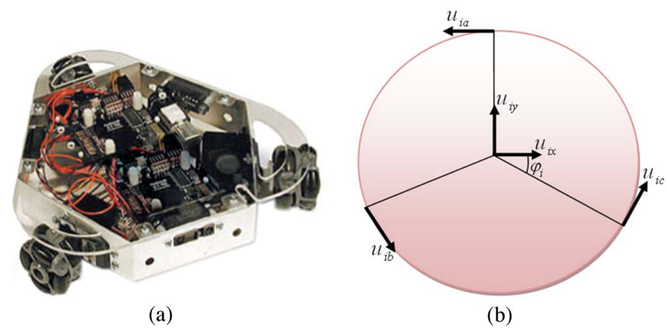

Fig. 9. PPRK platform used for experimental verification. (a) Real photograph. (b) Physical model.

Fig. 10. Captured photograph from the experimental test bed. (a) Initial positions. (b) Final positions.

The physical model of a single PPRK is shown in Fig. 9(b). Based on this physical model of the mobile robot, a simple 3 $\times$ 3 matrix can transform the main 2-D input (i.e., $u_i = [u_{ix}\ u_{iy}]^{\mathrm{T}}$) to the three omnidirectional wheel inputs of the ith robot (i.e., $[u_{ia}\ u_{ib}\ u_{ic}]^{\mathrm{T}}$)

$$\begin{bmatrix} u_{ia} \\ u_{ib} \\ u_{ic} \end{bmatrix} = \begin{bmatrix} \sin\phi_i & \cos\phi_i \\ -\sin\left(\frac{2\pi}{3}-\phi_i\right) & -\cos\left(\frac{2\pi}{3}-\phi_i\right) \\ -\cos\left(\frac{2\pi}{3}+\phi_i\right) & -\cos\left(\frac{2\pi}{3}+\phi_i\right) \end{bmatrix} \begin{bmatrix} u_{ix} \\ u_{iy} \end{bmatrix} \quad (47)$$

where ϕ_i represents the robot angle. The calculations required for (47) are implemented simply in the internal controller of each robot.

For this experimental verification, the same control scheme as presented in Section V-A is used. However, some parameters such as adaptation law and fuzzy membership functions are adopted to the new conditions. In addition, based on the robot mass, the matrix M in (16) is estimated as

$$M = \begin{bmatrix} 0.45 & 0 \\ 0 & 0.45 \end{bmatrix}.$$

The initial and final positions of robotic swarm are shown in Fig. 10(a) and (b), respectively. The captured data of robot motion trajectories are shown in Fig. 11(a), and the potential function decrement is shown in Fig. 11(b).

As expected, the motion trajectories of this simulation are very similar to the results of Example I in Fig. 2(a).

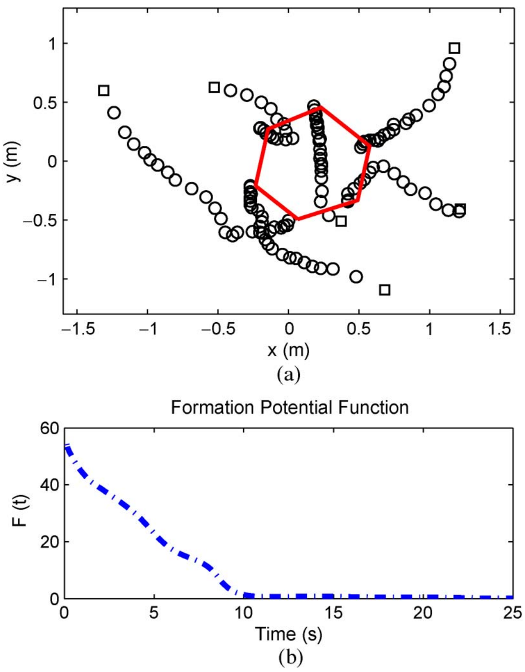

Fig. 11. Hexagonal formation of a real swarm of six robots. (a) Formation trajectory (the initial positions are indicated with square marks). (b) Formation potential.

VII. Conclusion

In this paper, the formation control problem of a class of multiagent systems with partially unknown dynamics has been investigated. On the basis of the Lyapunov stability theory, a novel decentralized adaptive fuzzy controller with corresponding parameter update law was developed, and the stability of the system was proved even in the case of external disturbances and input nonlinearities. All the theoretical results were verified by simulation examples, and good performance of the proposed controller was shown even in the case of agent failure and presence of measurement noises. Finally, a swarm of six real mobile robots were used to prove the applicability of the control scheme in real applications.

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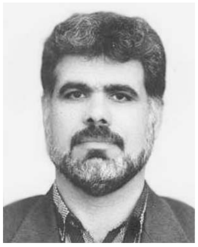
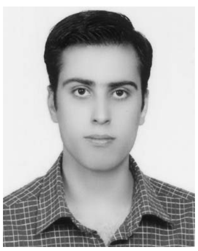
Bijan Ranjbar-Sahraei (M’09) was born in Shiraz, Iran, in 1986. He received the B.Sc. and M.Sc. degrees in control engineering from Shiraz University, Shiraz, in 2008 and 2011, respectively.

He is currently with the Department of Knowledge Engineering, Maastricht University, Maastricht, The Netherlands. He serves as an Associate Editor for the *Paladyn. Journal of Behavioral Robotics*. His research interests are swarm robotics and mobile sensor networks with emphasis on robust/adaptive control strategies.



Faridoon Shabaninia (M'81–SM'95) received the B.Sc. degree from the University of Washington, Seattle, the M.Sc. degree from San Jose State University, San Jose, CA, and the Ph.D. degree from New Mexico State University, Las Cruces, all in electrical engineering.

He is currently teaching in the Department of Power and Control Engineering, School of Electrical and Computer Engineering, Shiraz University, Shiraz, Iran, where he has been with department since 1998 and is exceptionally proud to have participated in the education of a couple of thousand students. He is involved in several research projects. He has authored over 60 journal and conference papers and five books. His professional interests include control systems, state estimation, simulation and modeling, and intelligent systems.



Alireza Nemati received the B.Sc. degree in mechanical engineering from Shiraz University, Shiraz, Iran, in 2008, and the M.Sc. degree in mechanical engineering and applied mechanics from Sharif University of Technology, Tehran, Iran, in 2010, where he is currently working toward the Ph.D. degree.

Since 2009, he has been the Internal Manager of the Center of Excellence in Design, Robotics, and Automation, Sharif University of Technology. His current research interests include swarm robotics and multiagent systems.

Sergiu-Dan Stan (M'07) received the Ph.D. degree in mechanical engineering from the Technical University of Cluj-Napoca, Cluj-Napoca, Romania.

He is currently with the Department of Mechatronics, Technical University of Cluj-Napoca, where he has led a number of national and international research grants in the area of mechatronics/robotics with applications in kinematics, control, design, virtual reality, and optimization as a University Collaborator or a Principal Investigator. He has published over 100 refereed articles in international journals, books, and conference proceedings.

Dr. Stan is member of the American Society of Mechanical Engineers, the European Mechanics Society, and the International Federation for the Promotion of Mechanism and Machine Science.