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Output feedback controller for hysteretic time-delayed MIMO nonlinear systems

An H^∞-based indirect adaptive interval type-2 fuzzy approach

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Abstract In this paper, an H^∞ output feedback controller is developed for a class of time-delayed MIMO nonlinear systems, containing backlash as an input nonlinearity. Particularly, a state observer is proposed to estimate unmeasurable states. The control law can be divided into two elements: An adaptive interval type-2 fuzzy part which approximates the uncertain model. The second part is an H^∞-based controller, which attenuates the effects of external disturbances and approximation errors to a prescribed level. Furthermore, the Lyapunov theorem is used to prove stability of proposed controller and its robustness to external disturbance, hysteresis input nonlinearity, and time varying time-delay. As an example, the designed controller is applied to address the tracking problem of 2-DOF robotic manipulator. Simulation results not only verify the robust properties but also in comparison with an existing method reveal the ability of the proposed controller to exclude the effects of unknown time varying time-delays and hysteresis input nonlinearity.

Keywords Interval type-2 fuzzy approximator · MIMO control · H_∞ control · Hysteretic systems · Robotic manipulator control

1 Introduction

In the last two decades, significant developments have been made in the theory of nonlinear feedback control. One of the traditional schemes for nonlinear control and almost the most efficient one is feedback linearization method. However, in this control method, the exact model of the system is assumed to be known to maintain sufficient control performance. To overcome this restrictive assumption, many control schemes based on intelligent techniques such as adaptive fuzzy systems have been developed, which makes an approximation of the unknown parts of the model [2, 7, 23, 32].

Generally, adaptive fuzzy controllers are categorized as direct and indirect approaches. In the direct one, an adaptive fuzzy system is applied to approximate the controller [15, 30]; on the other hand, in the indirect scheme an adaptive fuzzy system is used to approximate only the unknown parts of the model [24, 30].

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In principle, most of uncertain nonlinear systems might consist of noisy measurements and disturbances, while type-1 fuzzy approximators are proved to have some weaknesses in modeling and approximation of such uncertainties, as they use crisp rules [18, 29]. In an interval type-2 fuzzy logic system, to overcome this weakness, a collection of fuzzy antecedents and consequents of the rule-base are used in exchange for the type-1 crisp ones. Wide range of applications of type-2 fuzzy logic systems shows that it can be used when circumstances are too uncertain to design exact rules, such as when training data is corrupted by noise [1, 5, 14, 19, 20, 25].

While approximating the unknown parts of the model, some approximation errors and bounded external disturbances emerge. To eliminate the effects of these approximation errors and the bounded external disturbances various robust methods such as variable structure [21, 22] and H_∞ [31] techniques have been applied. Lin et al. [15] designed a direct MIMO H_∞ state feedback adaptive interval type-2 fuzzy controller to handle the training data, which was corrupted by noise and rule uncertainties. They proposed an H_∞ tracking technique to attenuate the effect of matching error and external disturbance to an arbitrary desired level. Hsiao et al. [9] proposed an interval type-2 fuzzy sliding-mode controller for a class of linear and nonlinear SISO systems. The proposed controller was a combination of the interval type-2 fuzzy logic control with Sliding Mode Control (SMC). Moreover, Lin et al. [16] proposed a Fuzzy Neural Network (FNN), based on the adaptive interval type-2 fuzzy systems, in which the modeling errors can be eliminated for a class of SISO time-delayed nonlinear systems. However, in practice, another main issue is input nonlinearities, which can degrade the performance of practical system and even worse may lead to instability of closed loop systems. The main contribution of this paper is to handle the mentioned practical limitations.

Hysteresis is an interesting phenomenon that occurs in a wide variety of physical systems (e.g., piezoelectric actuators and electromagnetic devices [10, 28]). Moreover, control of such systems is still a challenging open research topic in the literature [6, 11, 26]. As a mathematical description, the output of a hysteresis operator at time instant t depends on the input value at the same time (t) and some history values of input and output. This main property of hysteresis leads to a complicated behavior. To represent this behavior, various models of hysteresis such as Preisach [3], Bouc-Wen [11], Prandtl-Ishlinskii [4], and backlash [34], have been proposed and several adaptive techniques have been designed to address these uncertain nonlinear control problems [6, 11, 26, 34].

In this paper, an observer based Indirect Adaptive Interval Type-2 Fuzzy (IAIT2F) output feedback controller is proposed for a class of MIMO hysteretic systems which contain time varying time-delay.

The main features of proposed controller are listed as following:

- This paper employs an observer to estimate state variables.
- Two interval type-2 fuzzy systems are applied to approximate the unknown parts of the model.
- An H^∞-based controller is designed to attenuate the approximation error and external disturbances to a prescribed level.
- It is assumed that the control input is affected by a backlash-like hysteresis operator.
- The controller stability is proved in the presence of unknown time varying time-delays.

To the best of authors' knowledge, this is the first report of proposing a robust control technique for a MIMO nonlinear system containing both backlash-like hysteresis and time varying time-delayed dynamics.

The rest of this paper is organized as follows: Sect. 2 presents the problem formulation. Section 3 introduces the interval type-2 fuzzy logic systems. State-observer and the control design are discussed in Sect. 4. Simulation results are included in Sects. 5 and 6 provides the concluding remarks.

2 Problem formulation

Consider a class of MIMO nonlinear time-delayed systems, described by

$$\begin{aligned} y_i^{(r_i)} &= f_{i1}\big(\underline{x}(t)\big) + f_{i2}\big(\underline{x}(t), \underline{x}\big(t-\tau(t)\big)\big) \\ &\quad + \sum_{j=1}^{m} g'_{ij}\big(\underline{x}(t)\big) P[u_j](t) - d_i, \\ i &= 1, 2, \ldots, m \end{aligned} \tag{1}$$

where P denotes the backlash-like hysteresis operator, $\underline{x} = [x_1, \dot{x}_1, \ldots, x_1^{r_1}, \ldots, x_m, \dot{x}_m, \ldots, x_m^{r_m}]^T \in \mathbb{R}^n$

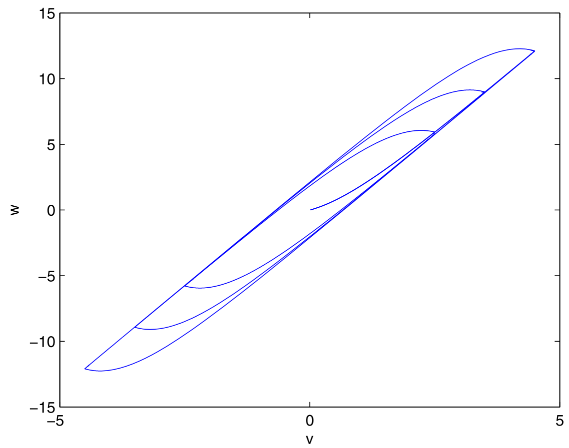

Fig. 1 Backlash-like hysteresis

is the state vector, $u = [u_1, \dots, u_m]^T \in \mathbb{R}^m$ is the input, $y = [y_1, \dots, y_m]^T \in \mathbb{R}^m$ is the output and d_i represents the disturbance term. Moreover, $r_1, \dots, r_m$ are the subsystems relative degrees and $\tau(t)$ is a time varying time-delay which satisfies the following relations:

$$\tau(t) \le \tau_0, \tag{2}$$

$$\dot{\tau}(t) \le \tau_1 < 1. \tag{3}$$

Remark 1 A backlash-like hysteresis operator $\omega(t) = P[v](t)$ can be described by [26]

$$\frac{d\omega}{dt} = \alpha \left|\frac{dv}{dt}\right| (cv - \omega) + B_1 \frac{du}{dt}, \tag{4}$$

where $v \in \mathbb{R}$ is the control input, α, c and B_1 are some arbitrary constants which satisfy $c > 0$ and $c > B_1$.

The explicit solution of (4) can be derived as

$$\omega(t) = cv(t) + z(v), \tag{5}$$

where

$$z(v) = [w_0 - cv_0] e^{-\alpha(v - v_0)\operatorname{sgn}(\dot{v})} + e^{-\alpha v \operatorname{sgn}(\dot{v})} \int_{v_0}^{u} [B_1 - c] e^{\alpha \zeta \operatorname{sgn}(\dot{v})} \, d\zeta. \tag{6}$$

The model characteristic is shown in Fig. 1 for input signal $v(t) = 6.5 \sin(2.3t)$ and $\alpha = 1$, $c = 3.1635$, $B_1 = 0.345$ and $\omega(0) = 0$.

Using (5), the system dynamic in (1) can be rewritten in the following form:

$$y_i^{(r_i)} = f_{i1}\big(\underline{x}(t)\big) + f_{i2}\big(\underline{x}(t), \underline{x}\big(t - \tau(t)\big)\big) + \sum_{j=1}^{m} g_{ij}\big(\underline{x}(t)\big) u_j(t) + g'_{ij}\big(\underline{x}(t)\big) z(u_j) - d_i$$
$$i = 1, 2, \dots, m \tag{7}$$

where

$$g_{ij} = c g'_{ij}. \tag{8}$$

It is assumed that $f_{i1}(\underline{x}(t))$ and $g_{ij}(\underline{x}(t))$ can be written as

$$\begin{aligned} f_{i1}\big(\underline{x}(t)\big) &= f_{i1,no}\big(\underline{x}(t)\big) + \Delta f_{i1}\big(\underline{x}(t)\big) \\ g_{ij}\big(\underline{x}(t)\big) &= g_{ij,no}\big(\underline{x}(t)\big) + \Delta g_{ij}\big(\underline{x}(t)\big). \end{aligned} \tag{9}$$

In the above equations, $f_{i1,no}(\underline{x}(t))$ and $g_{ij,no}(\underline{x}(t))$ are the known nominal parts and $\Delta f_{i1}(\underline{x}(t))$ and $\Delta g_{ij}(\underline{x}(t))$ represent the uncertain nonlinearities of $f_{i1}(\underline{x}(t))$ and $g_{ij}(\underline{x}(t))$. Equation (1) can be expressed in a normal

$$\begin{cases} \dot{\underline{x}}_i = A_i \underline{x}_i + B_i \big[f_{i1}\big(\underline{x}(t)\big) \\ \qquad + f_{i2}\big(\underline{x}(t), \underline{x}\big(t - \tau(t)\big)\big) \\ \qquad + G_i\big(\underline{x}(t)\big) u + G'_i\big(\underline{x}(t)\big) Z(u) - d_i \big] \\ y_i = C_i^T \underline{x}_i, \quad i = 1, 2, \dots, m \end{cases} \tag{10}$$

in which $Z(u) \triangleq [z(u_1), z(u_2), \dots, z(u_m)]$ and

$$A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{r_i \times r_i}, \qquad B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_i \times 1},$$

$$\begin{aligned} C_i^T &= [1\ 0\ \cdots\ 0]_{1 \times r_i}, \\ G_i\big(\underline{x}(t)\big) &= G_{i,no}\big(\underline{x}(t)\big) + \Delta G_i\big(\underline{x}(t)\big), \\ G_{i,n_o}\big(\underline{x}(t)\big) &= [g_{i1,no}, \dots, g_{im.no}], \\ \Delta G_i\big(\underline{x}(t)\big) &= [\Delta g_{i1}, \dots, \Delta g_{im}], \\ G'_i\big(\underline{x}(t)\big) &= [g'_{i1}, \dots, g'_{im}]. \end{aligned}$$

Using (9) and (10), it can be concluded that (1) can be rewritten in a compact form as

$$\begin{cases} \dot{\underline{x}} = A + B \big[F_{o1}\big(\underline{x}(t)\big) + \Delta F_1\big(\underline{x}(t)\big) \\ \qquad + F_2\big(\underline{x}(t), \underline{x}\big(t - \tau(t)\big) \\ \qquad + \big[G_o\big(\underline{x}(t)\big) + \Delta G\big(\underline{x}(t)\big)\big] u \\ \qquad + G'\big(\underline{x}(t)\big) Z(u) - d \big] \\ y = C^T \underline{x} \end{cases} \tag{11}$$

with notations, defined as

$$F_{o1}(\underline{x}(t)) = [f_{11,no}(\underline{x}(t)), \dots, f_{m1,no}(\underline{x}(t))]^T,$$

$$\Delta F_1(\underline{x}(t)) = [\Delta f_{11}(\underline{x}(t)), \dots, \Delta f_{m1}(\underline{x}(t))]^T,$$

$$\begin{aligned} &F_2(\underline{x}(t), \underline{x}(t-\tau(t))) \\ &\quad = [f_{12}(\underline{x}(t), \underline{x}(t-\tau(t))), \dots, \\ &\qquad f_{m2}(\underline{x}(t), \underline{x}(t-\tau(t)))]^T, \end{aligned}$$

$$G_o(\underline{x}(t)) = [G_{1,no}(\underline{x}(t)), \dots, G_{m,no}(\underline{x}(t))]^T,$$

$$\Delta G(\underline{x}(t)) = [\Delta G_1(\underline{x}(t)), \dots, \Delta G_m(\underline{x}(t))]^T,$$

$$G'(\underline{x}(t)) = [G'_1(\underline{x}(t)), \dots, G'_m(\underline{x}(t))]^T$$

$$A = \mathrm{diag}[A_1, \dots, A_m],$$

$$B = \mathrm{diag}[B_1, \dots, B_m],$$

$$C = \mathrm{diag}[C_1, \dots, C_m],$$

$$d = [d_1, \dots, d_m]^T,$$

and following assumptions are held.

Assumption 1 The matrix $G(\underline{x}(t))$ is invertible.

Assumption 2 The function vector $F_2(\underline{x}(t), \underline{x}(t-\tau(t)))$ satisfies the following inequality

$$\|F_2\| \le \alpha \|x(t)\| + \beta \|x(t-\tau(t))\| \tag{12}$$

where α and β are unknown constants.

Assumption 3 K_o and K_c are chosen such that for given positive definite matrices Q_1 and Q_2, there exist positive definite matrices P_1 and P_2 which satisfy

$$(A - BK_C^T)^T P_1 + P_1(A - BK_C^T) = -Q_1 \tag{13}$$

and

$$\begin{cases} (A - K_oC^T)^T P_2 + P_2(A - K_oC^T) = -Q_2 \\ P_2B = C \end{cases} \tag{14}$$

which concludes to the satisfaction of Hurwitz criteria by $(A - BK_C{}^T)$ and the SPR criteria by $[A - K_oC^T, B, C]$.

Consider $y_m = [y_{1m}, y_{2m}, \dots, y_{mm}]^T$ as the desired reference signal and define

$$\underline{x}_d = [y_{1m}, \dots, y_{1m}^{(r_1-1)}, \dots, y_{mm}, \dots, y_{mm}^{(r_m-1)}]^T.$$

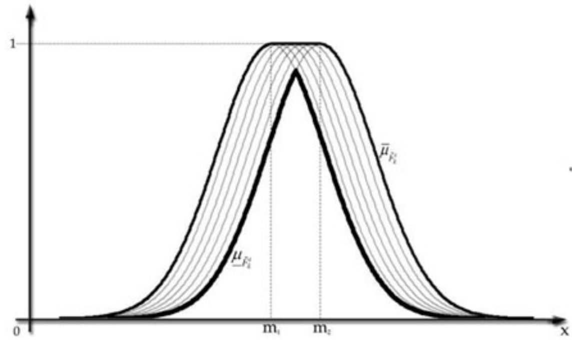

Fig. 2 Interval type-2 fuzzy set (IT2FS)

The control problem is to design a control law which forces the system output to track the reference signal y_m.

Assumption 4 The desired trajectory x_d is continuous and differentiable.

Assumption 5 There exist a constant ℓ for every compact set $U \subset \mathbb{R}^n$ such that

$$\|G'(\underline{x}(t))\| \le \ell \quad \forall x \in U. \tag{15}$$

In the next section, a brief introduction on interval type-2 fuzzy logic systems will be presented, and Sect. 4 introduces the proposed control scheme.

3 Interval type-2 fuzzy logic system

Fuzzy Logic Systems (FLSs) are known as the universal approximators and have various applications in identification and control design. A type-1 fuzzy system consists of four major parts: fuzzifier, rule base, inference engine, and defuzzifier. A type-2 fuzzy system has a similar structure, but one of the major differences can be seen in the rule base part, where a type-2 rule base has antecedents and consequents using type-2 Fuzzy Sets (T2FS). In a T2FS, a Gaussian function with a known standard deviation is chosen, while the mean (m) varies between m_1 and m_2. Therefore, a uniform weighting is assumed to represent a footprint of uncertainty as shaded in Fig. 2. Because of using such a uniform weighting, we name the T2FS as an Interval Type-2 Fuzzy Set (IT2FS).

Utilizing a rule base which consists of IT2FSs, the output of the inference engine will also be a T2FS,

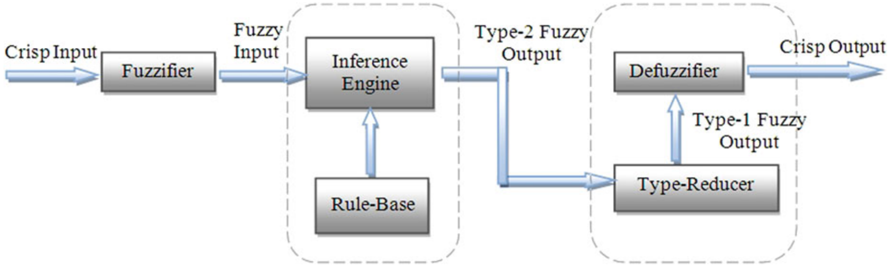

Fig. 3 Main structure of an interval type-2 FLS (IT2FLS)

and, therefore, we need a type-reducer to convert it to a type-1 fuzzy set before defuzzification can be carried out. Figure 3 shows the main structure of an interval type-2 FLS.

By using singleton fuzzification, the singleton inputs are fed into the inference engine. Combining the fuzzy if-then rules, the inference engine maps the singleton input $x = [x_1, x_2, \ldots, x_n]$ into a type-2 fuzzy set as the output. A typical form of an if-then rule is

$$R_i = \text{if } x_1 \text{ is } \tilde{F}_1^i \text{ and } x_2 \text{ is } \tilde{F}_2^i \text{ and} \cdots \text{and } x_n \text{ is } \tilde{F}_n^i \text{ then } \tilde{G}^i \tag{16}$$

where $\tilde{F}_k^i$ s are the antecedents $(k = 1, 2, \ldots, n)$ and $\tilde{G}^i$ is the consequent of the ith rule. The sup-star method is chosen among various inference methods and the first step to evaluate the firing set for ith rule is

$$F^i(\underline{x}) = \prod_{k=1}^{n} \mu_{\tilde{F}_k^i}(x_k). \tag{17}$$

As all of the $\tilde{F}_k^i$s are IT2FSs, so $F^i(\underline{x})$ can be written as $F^i(\underline{x}) = [\underline{f}^i(\underline{x})\ \underline{f}^i(\overline{x})]$ where

$$\underline{f}^i(\underline{x}_i) = \prod_{k=1}^{n} \underline{\mu}_{\tilde{F}_k^i}(x_k), \tag{18}$$

$$\overline{f}^i(\underline{x}_i) = \prod_{k=1}^{n} \overline{\mu}_{\tilde{F}_k^i}(x_k). \tag{19}$$

The terms $\underline{\mu}_{\tilde{F}_k^i}$ and $\bar{\mu}_{\tilde{F}_k^i}$ are the lower and upper membership functions, respectively (Fig. 2).

In the next step, the firing set $F^i(\underline{x})$ is combined with the ith consequent using the product t-norm to produce the type-2 output fuzzy set. The type-2 output fuzzy sets are then fed into the type reduction part. The structure of type reducing procedure is combined with the defuzzification procedure, which uses Center of

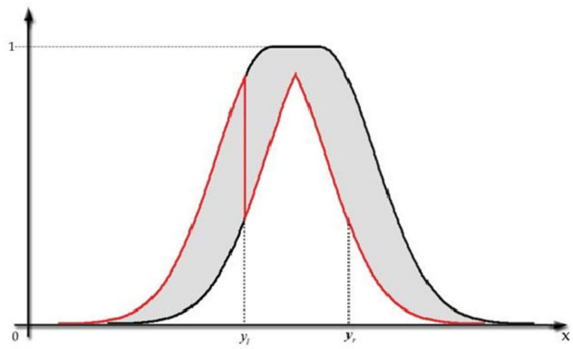

Fig. 4 Computing right and left centroids for an IT2FS

Sets (COS) method. First, the left and right centroids of each rule consequent is computed using Karnik–Mendel (KM) algorithm [17] as shown in Fig. 4. Let us call it $[y_l\ y_r]$.

The firing sets $F^i(\underline{x}) = [\underline{f}^i(\underline{x})\, \bar{f}^i(\underline{x})]$ computed in inference engine are combined with the left and right centroids of consequents, and then the defuzzified output is evaluated by finding the solutions of the following optimization problems:

$$y_l(\underline{x}) = \min_{\forall f^k \in \{\underline{f}^k\, \bar{f}^k\}} \left(\sum_{k=1}^{M} y_l^k f^k(\underline{x}) \Big/ \sum_{k=1}^{M} f^k(\underline{x}) \right), \tag{20}$$

$$y_r(\underline{x}) = \max_{\forall f^k \in \{\underline{f}^k\, \bar{f}^k\}} \left(\sum_{k=1}^{M} y_r^k f^k(\underline{x}) \Big/ \sum_{k=1}^{M} f^k(\underline{x}) \right). \tag{21}$$

Define $f_l^k(\underline{x})$ and $f_r^k(\underline{x})$ as the functions that are used to solve (20) and (21), respectively, and let

$$\xi_l^k(\underline{x}) = f_l^k(\underline{x}) \Big/ \sum_{k=1}^{M} f_l^k(\underline{x})$$

and

$$\xi_r^k(\underline{x}) = f_r^k(\underline{x}) \Big/ \sum_{k=1}^{M} f_r^k(\underline{x});$$

then (20) and (21) can be rewritten as

$$\begin{aligned} y_l(\underline{x}) &= \sum_{k=1}^{M} y_l^k f_l^k(\underline{x}) \Big/ \sum_{k=1}^{M} f_l^k(\underline{x}) \\ &= \sum_{k=1}^{M} y_l^k \xi_l^k(\underline{x}) \\ &= \theta_l^T \xi_l(\underline{x}) \end{aligned} \tag{22}$$

and

$$\begin{aligned} y_r(\underline{x}) &= \sum_{k=1}^{M} y_r^k f_r^k(\underline{x}) \Big/ \sum_{k=1}^{M} f_r^k(\underline{x}) \\ &= \sum_{k=1}^{M} y_r^k \xi_r^k(\underline{x}) \\ &= \theta_r^T \xi_r(\underline{x}) \end{aligned} \tag{23}$$

where

$$\xi_l(\underline{x}) = \left[\xi_l^1(\underline{x})\ \xi_l^2(\underline{x}) \cdots \xi_l^M(\underline{x})\right]$$

and

$$\xi_r(\underline{x}) = \left[\xi_r^1(\underline{x})\ \xi_r^2(\underline{x}) \cdots \xi_r^M(\underline{x})\right]$$

are the fuzzy basis functions, and

$$\theta_l(\underline{x}) = \left[y_l^1(\underline{x})\ y_l^2(\underline{x}) \cdots y_l^M(\underline{x})\right]$$

and

$$\theta_r(\underline{x}) = \left[y_r^1(\underline{x})\ y_r^2(\underline{x}) \cdots y_r^M(\underline{x})\right]$$

are the adjustable parameters.

Finally, the crisp value is obtained by the defuzzificatin procedure as

$$\begin{aligned} y(\underline{x}) &= \frac{1}{2}\big(y_l(\underline{x}) + y_r(\underline{x})\big) = \frac{1}{2}\big(\theta_l^T \xi_l(\underline{x}) + \theta_r^T \xi_r(\underline{x})\big) \\ &= \frac{1}{2}\theta^T \xi(\underline{x}), \end{aligned} \tag{24}$$

where $\theta = [\theta_l^T\ \theta_r^T]^T$ and $\xi = [\xi_l^T\ \xi_r^T]^T$.

4 State-observer and controller design

In order to propose two fuzzy approximators based on interval type-2 FLS, the unknown functions $\Delta f_{i1}(\underline{x}(t))$ and $\Delta g_{ij}(\underline{x}(t))$ can be approximated as:

$$\hat{f}_{i1}\big(\underline{x}(t)|\theta_{f_i}\big) = \theta_{f_i}^T \xi_1\big(\underline{x}(t)\big) \quad i = 1, \dots, m, \tag{25}$$

$$\hat{g}_{ij}\big(\underline{x}(t)|\theta_{g_{ij}}\big) = \theta_{g_{ij}}^T \xi_2\big(\underline{x}(t)\big) \quad i, j = 1, \dots, m. \tag{26}$$

Therefore, $\Delta F_1(\underline{x}(t))$ and $\Delta G(\underline{x}(t))$ can be approximated in a compact form as

$$\hat{F}_1\big(\underline{x}(t)|\Theta_1\big) = \Gamma\big(\underline{x}(t)\big)^T \Theta_1, \tag{27}$$

and

$$\hat{G}\big(\underline{x}(t)|\Theta_2\big) = \Phi\big(\underline{x}(t)\big)^T \Theta_2, \tag{28}$$

where

$$\begin{aligned} &\Theta_1 = [\theta_{f_1}, \theta_{f_2}, \dots, \theta_{f_m}]^T \in R^{mp}, \\ &\Theta_2 = \begin{bmatrix} \theta_{g_{11}} & \cdots & \theta_{g_{1m}} \\ \vdots & \ddots & \vdots \\ \theta_{g_{m1}} & \cdots & \theta_{g_{mm}} \end{bmatrix} \in R^{mp \times m}, \\ &\theta_{f_1} = [\theta_{1f_1}, \theta_{2f_1}, \dots, \theta_{pf_1}] \in R^{1 \times P}, \\ &\theta_{g_{ij}} = [\theta_{1g_{ij}}, \theta_{2g_{ij}}, \dots, \theta_{pg_{ij}}]^T \in R^{p}, \\ &\Gamma\big(\underline{x}(t)\big) = \mathrm{diag}\big[\xi_1\big(\underline{x}(t)\big), \dots, \xi_1\big(\underline{x}(t)\big)\big] \in R^{mp \times m}, \\ &\varphi\big(\underline{x}(t)\big) = \mathrm{diag}\big[\xi_2\big(\underline{x}(t)\big), \dots, \xi_2\big(\underline{x}(t)\big)\big] \in R^{mp \times m}. \end{aligned}$$

Consider the adaptive state observer as

$$\begin{aligned} \dot{\hat{\underline{x}}} &= A\hat{\underline{x}} + B\big[F_{o1} + \hat{F}_1\big(\hat{\underline{x}}|\Theta_1\big) + \big(G_o + \hat{G}\big(\hat{\underline{x}}|\Theta_2\big)\big)u \\ &\quad - u_{\inf} - u_a - u_s\big] + K_O\big(y - C^T\hat{\underline{x}}\big) \\ &\quad + \frac{1}{2}B\hat{\theta}_0\big(y - C^T\hat{\underline{x}}\big), \end{aligned} \tag{29}$$

$$\hat{y} = C^T\hat{\underline{x}}, \tag{30}$$

where u and u_{inf} denote the fuzzy controller and robust controller, respectively. In addition, u_a and u_s are designed as compensation control signals. K_o is chosen according to Assumption 1. Moreover, $\hat{\theta}_0$ is updated by the adaptation law

$$\dot{\hat{\theta}}_0 = \gamma_0 \big\|y - C^T\hat{\underline{x}}\big\|^2. \tag{31}$$

Applying (11) and (29), the observer error dynamic $\underline{e} = \underline{x} - \hat{\underline{x}}$ can be calculated as

$$\dot{\underline{e}} = \left(A - K_O C^T\right)\underline{e} + B\left[\left(\Delta F_1 - \hat{F}_1\right) + \left(\Delta G - \hat{G}\right)u + G'\left(\underline{x}(t)\right)Z(u) + u_{\inf} + u_a + u_s - d\right] + B\left(F_2 - \frac{1}{2}\hat{\theta}_0 C^T \underline{e}\right). \tag{32}$$

Definition 1 The minimum approximation error is defined as

$$\omega \triangleq \Delta F_1\left(\underline{x}(t)\right) - \hat{F}_1\left(\hat{\underline{x}}(t)|\Theta_1^*\right) + \left[\Delta G\left(\underline{x}(t)\right) - \hat{G}\left(\hat{\underline{x}}(t)|\Theta_2^*\right)\right]u \tag{33}$$

with Θ_1^* and Θ_2^* defined as the optimal coefficients, and

$$\Theta_1^* \triangleq \arg\min_{\Theta_1 \in \Omega_1}\left[\sup\left\| \hat{F}_1\left(\hat{\underline{x}}(t)|\Theta_1\right) - \Delta F_1\left(\underline{x}(t)\right)\right\|\right], \tag{34}$$

$$\Theta_2^* \triangleq \arg\min_{\Theta_2 \in \Omega_2}\left[\sup\left\| \hat{G}\left(\hat{\underline{x}}(t)|\Theta_1\right) - \Delta G\left(\underline{x}(t)\right)\right\|\right] \tag{35}$$

where Ω_1 and Ω_2 are proper compact sets defined as

$$\Omega_1 = \left\{\Theta_1 \in \mathbb{R}^{mp} \mid \|\Theta_1\| \le D_1\right\}, \tag{36}$$

$$\Omega_2 = \left\{\Theta_2 \in \mathbb{R}^{mp\times m} \mid \|\Theta_2\| \le D_2\right\}. \tag{37}$$

Definition 2 R is defined such that $R^{-1} \ge 2\rho^2 I$ to guarantee the existence of solution for the Riccati equation

$$\left(A - K_O C^T\right)^T P_2 + P_2\left(A - K_O C^T\right) + Q_2 - 2C\left(R^{-1} - \frac{1}{2\rho^2}I\right)C^T = 0, \tag{38}$$

in which ρ is a selective weight.

Using Definition 1 and notations $\tilde{\Theta}_1 = \Theta_1 - {\Theta_1}^*$ and $\tilde{\Theta}_2 = \Theta_2 - {\Theta_2}^*$, (32) can be rewritten as

$$\dot{\underline{e}} = \left(A - K_O C^T\right)\underline{e} + B\left[\Gamma^T \hat{\underline{x}}(t)\tilde{\Theta}_1 + \left(\varphi^T \hat{\underline{x}}(t)\tilde{\Theta}_2 u + G'\left(\underline{x}(t)\right)Z(u) + u_{\inf} + u_a + u_s + \omega - d\right]\right. + B\left(F_2 - \frac{1}{2}\hat{\theta}_0 C^T \underline{e}\right). \tag{39}$$

Now, consider the following control and adaptation laws:

$$u = \left[\hat{G}\left(\hat{\underline{x}}(t)|\Theta_2\right) + G_0\right]^{-1}\Bigg[- \hat{F}_1\left(\hat{\underline{x}}(t)|\Theta_1\right) - F_{01} + y_m^{(r)} + K_C^T \hat{\underline{e}} + u_{\inf} + u_a + u_s - \frac{1}{2}\hat{\theta}_0\left(y - C^T\hat{\underline{x}}\right)\Bigg], \tag{40}$$

$$u_{\inf} = -R^{-1}C^T\underline{e}, \tag{41}$$

$$u_a = -C^T\underline{e}\|Y_m\|, \tag{42}$$

$$u_s = K_0^T P_1 \hat{\underline{e}}^T, \tag{43}$$

$$\dot{\Theta}_1 = \begin{cases} -\gamma_1\Gamma(\hat{\underline{x}}(t))C^T\underline{e} \\ \quad \text{if } \{\{\|\Theta_1\| \le D_1\} \text{ or} \\ \quad \{\|\Theta_1\| = D_1 \text{ and} \\ \quad \Theta_1^T\Gamma(\hat{\underline{x}}(t))C^T\underline{e} \ge 0\}\} \\ \mathrm{Proj}(-\gamma_1\Gamma(\hat{\underline{x}}(t))C^T\underline{e}) \\ \quad \text{if } \{\|\Theta_1\| = D_1 \text{ and} \\ \quad \Theta_1^T\Gamma(\hat{\underline{x}}(t))C^T\underline{e} < 0\} \end{cases} \tag{44}$$

$$\dot{\Theta}_2 = \begin{cases} -\gamma_2\varphi(\hat{\underline{x}}(t))C^T\underline{e}u^T \\ \quad \text{if } \{\{\|\Theta_1\| \le D_2\} \text{ or} \\ \quad \{\||\Theta_2\| = D_2 \text{ and} \\ \quad \mathrm{tr}(\gamma_2\varphi(\hat{\underline{x}}(t))C^T\underline{e}u^T\Theta^T) \ge 0\}\} \\ \mathrm{Proj}(-\gamma_2\varphi(\hat{\underline{x}}(t))C^T\underline{e}u^T) \\ \quad \text{if } \{\|\Theta_2\| = D_2 \text{ and} \\ \quad \mathrm{tr}(\gamma_2\varphi(\hat{\underline{x}}(t))C^T\underline{e}u^T{\Theta_2}^T) < 0\} \end{cases} \tag{45}$$

where $\gamma_1 > 0, \gamma_2 > 0$ are selective adaptation gains, $\hat{\underline{e}} = \underline{x}_d - \hat{\underline{x}}$ is the tracking error and

$$\mathrm{Proj}\left(-\gamma_1\Gamma\left(\hat{\underline{x}}(t)\right)C^T\underline{e}\right) = -\gamma_1\Gamma\left(\hat{\underline{x}}(t)\right)C^T\underline{e} + \Theta_1^T\gamma_1\Gamma\left(\hat{\underline{x}}(t)\right)C^T\underline{e}\frac{\Theta_1}{\|\Theta_1\|^2}, \tag{46}$$

$$\mathrm{Proj}\left(-\gamma_2\varphi\left(\hat{\underline{x}}(t)\right)C^T\underline{e}u^T\right) = -\gamma_2\varphi\left(\hat{\underline{x}}(t)\right)C^T\underline{e}u^T + \mathrm{tr}\left(\gamma_2\varphi\left(\hat{\underline{x}}(t)\right)C^T\underline{e}u^T{\Theta_2}^T\right)\frac{\Theta_2}{\|\Theta_2\|^2}. \tag{47}$$

The following theorem shows that the designed controller guarantees the stability and robustness of closed loop system.

Theorem 1 *Consider a class of MIMO systems as shown in (1) and assume that only output variables are measurable. If Assumptions 1, 2, 3, 4, and 5 are satisfied, the control and adaptation laws (40)–(45) guarantee the global stability and robustness of closed loop system, i.e.,* $\underline{x}, \hat{\underline{x}}, \underline{e}, \hat{\underline{e}}, u \in L_\infty$ *and the following* H^∞

criterion will be satisfied:

$$\frac{1}{2}\int_0^T \underline{\hat{e}}^T Q_1' \underline{\hat{e}}\, dt + \frac{1}{2}\int_0^T \underline{e}^T Q_2' \underline{e}\, dt \leq V(0) + \frac{\rho^2}{2}\int_0^T (\omega - d)^T(\omega - d)\, dt \tag{48}$$

Moreover, if $(\omega - d) \in L^2$, *then it can be concluded that* $\underline{\hat{e}}, \underline{e} \in L^2$.

Proof From (6), it can be easily proved that [26]

$$\lim_{x\to\infty} z(u_i) = -\frac{c - B_1}{\alpha}, \quad i = 1, 2, \ldots, m, \tag{49}$$

$$\lim_{x\to-\infty} z(u_i) = \frac{c - B_1}{\alpha}, \quad i = 1, 2, \ldots, m. \tag{50}$$

Therefore, regarding Assumption 5, (39) can be rewritten as

$$\dot{\underline{e}} = (A - K_O C^T)\underline{e} + B\big[\Gamma^T(\hat{\underline{x}}(t)|\tilde{\Theta}_1) + (\varphi^T(\hat{\underline{x}}(t)|\tilde{\Theta}_2)u + u_{\inf} + u_a + u_s - d'\big] \tag{51}$$

in which $d' = d - G'(\underline{x}(t))Z(u)$ is a bounded unknown term.

By defining $y_m{}^{(r)} = [y_{1m}{}^{r_1}, y_{2m}{}^{r_2}, \ldots, y_{mm}{}^{r_m}]$, the following relation can be achieved:

$$\dot{\underline{\hat{e}}} = A\underline{x}_d + By_m^{(r)} - \dot{\underline{\hat{x}}}, \tag{52}$$

and with regard to (52), applying (40) to (29), leads to the tracking error dynamic

$$\dot{\underline{\hat{e}}} = (A - BK_C^T)\underline{\hat{e}} - K_O C^T \underline{e}. \tag{53}$$

Let us define a Lyapunov function as

$$V = \frac{1}{2}\underline{\hat{e}}^T P_1 \underline{\hat{e}} + \frac{1}{2}\underline{e}^T P_2 \underline{e} + \frac{\gamma}{1-\tau_1}\int_{t-\tau(t)}^{t}\big(\underline{e}^T(\lambda)\underline{e}(\lambda) + \underline{\hat{e}}^T(\lambda)\underline{\hat{e}}(\lambda)\big)\, d\lambda + \frac{1}{2\gamma_0}\tilde{\theta}_0^2 + \frac{1}{2\gamma_1}\tilde{\Theta}_1^T\tilde{\Theta}_1 + \frac{1}{2\gamma_2}\mathrm{tr}(\tilde{\Theta}_2^T\tilde{\Theta}_2), \tag{54}$$

in which

$$\tilde{\theta}_0 = \theta_0 - \theta_0^*. \tag{55}$$

Time derivative of V is computed as:

$$\begin{aligned}\dot{V} \leq{}& \frac{1}{2}\underline{\hat{e}}^T\big[P_1(A - BK_C) + (A - BK_C)^T P_1\big]\underline{\hat{e}} \\ &- \underline{\hat{e}} P_1 K_o C^T \underline{e} \\ &+ \frac{1}{2}\underline{e}^T\big[(A - K_o C^T)P_2 + P_2(A - K_o C^T)^T\big]\underline{e} \\ &+ \underline{e}^T P_2 B\Big[\Gamma^T(\hat{\underline{x}}(t))\tilde{\Theta}_1 + \varphi^T(\hat{\underline{x}}(t))\tilde{\Theta}_2 u \\ &+ (\omega - d) + u_{\inf} + u_s + u_a + F_2 - \frac{1}{2}\hat{\theta}_0 C^T\underline{e}\Big] \\ &+ \frac{\gamma}{1-\tau_1}\big(\|\underline{e}(t)\| + \|\underline{\hat{e}}(t)\|\big) \\ &- \gamma\big(\|\underline{e}(t - \tau(t))\| + \|\underline{\hat{e}}(t - \tau(t))\|\big) \\ &+ \frac{1}{\gamma_0}\dot{\hat{\theta}}_0\hat{\theta}_0 + \frac{1}{\gamma_1}\dot{\hat{\Theta}}_1{}^T\tilde{\Theta}_1 \\ &+ \frac{1}{\gamma_2}\mathrm{tr}(\dot{\hat{\Theta}}_2{}^T\tilde{\Theta}_2).\end{aligned} \tag{56}$$

Regarding Assumption 2, we have

$$\begin{aligned}&\|F_2\| \leq \alpha\|x(t)\| + \beta\|x(t - \tau(t))\| \\ \Rightarrow\quad &\|F_2\| \leq \alpha\|\underline{e}(t)\| + \alpha\|\underline{\hat{e}}(t)\| + \alpha\|Y_m\| \\ &\quad + \beta\|\underline{e}(t - \tau(t))\| + \beta\|\underline{\hat{e}}(t - \tau(t))\| \\ &\quad + \beta\|Y_m\|.\end{aligned} \tag{57}$$

Using Young's inequality [13], it can be concluded that

$$\begin{aligned}\underline{e}^T P_2 B F_2 \leq{}& \frac{2}{\gamma}(\alpha^2 + \beta^2)\|\underline{e}^T P_2 B\|^2 \\ &+ \gamma\big(\|\underline{e}(t)\|^2 + \|\underline{\hat{e}}(t)\|^2 + \|\underline{e}(t - \tau(t))\|^2 \\ &+ \|\underline{\hat{e}}(t - \tau(t))\|^2\big) + \|\underline{e}^T C\|\|Y_m\|\end{aligned} \tag{58}$$

where γ is a positive constant. According to Assumption 3 and by using (57), (56) can be simplified as

$$\begin{aligned}\dot{V} \leq{}& -\frac{1}{2}\underline{\hat{e}}^T Q_1\underline{\hat{e}} - \frac{1}{2}\underline{e}^T Q_2\underline{e} - \underline{\hat{e}} P_1 K_o C^T\underline{e} \\ &+ \underline{e}^T P_2 B\Gamma^T(\hat{\underline{x}}(t))\tilde{\Theta}_1 + \underline{e}^T P_2 B\varphi^T(\hat{\underline{x}}(t))\tilde{\Theta}_2 u \\ &+ \underline{e}^T P_2 B(\omega - d) + \underline{e}^T P_2 B(u_{\inf} + u_s + u_a) \\ &+ \gamma\big(\|\underline{e}(t)\|^2 + \|\underline{\hat{e}}(t)\|^2\big) \\ &+ \frac{\gamma}{1-\tau_1}\big(\|\underline{e}(t)\| + \|\underline{\hat{e}}(t)\|\big) \\ &- \frac{1}{2}\hat{\theta}_0\underline{e}^T CC^T\underline{e} + \|\underline{e}^T C\|\|Y_m\|\end{aligned}$$

$$+\frac{2}{\gamma}(\alpha^2+\beta^2)\|\underline{e}^T C\|^2 + \frac{1}{\gamma_0}\tilde{\theta}_0\dot{\hat{\theta}}_0 + \frac{1}{\gamma_1}\dot{\hat{\Theta}}_1{}^T\tilde{\Theta}_1 + \frac{1}{\gamma_2}\mathrm{tr}(\dot{\hat{\Theta}}_2{}^T\tilde{\Theta}_2). \quad (59)$$

For the sake of simplicity by defining $\theta_0^* \triangleq \frac{4}{\gamma}(\alpha^2+\beta^2)$, and using (41)–(43), (59) can be rewritten as

$$\begin{aligned}\dot{V} \le & -\frac{1}{2}\underline{\hat{e}}^T\left[Q_1-\left(\gamma+\frac{\gamma}{1-\tau_1}\right)\right]\underline{\hat{e}} \\ & -\frac{1}{2}\underline{e}^T\left[Q_2-\left(\gamma+\frac{\gamma}{1-\tau_1}\right)\right]\underline{e} \\ & -\underline{e}^T C\frac{1}{2}(\hat{\theta}_0-\theta_0^*)C^T\underline{e}+\underline{e}^T P_2 B\Gamma^T(\underline{\hat{x}}(t))\tilde{\Theta}_1 \\ & +\underline{e}^T P_2 B\varphi^T(\underline{\hat{x}}(t))\tilde{\Theta}_2 u+\frac{\rho^2}{2}(\omega-d)^T(\omega-d) \\ & +\frac{1}{\gamma_0}\tilde{\theta}_0\dot{\hat{\theta}}_0+\frac{1}{\gamma_1}\dot{\hat{\Theta}}_1{}^T\tilde{\Theta}_1+\frac{1}{\gamma_2}\mathrm{tr}(\dot{\hat{\Theta}}_2{}^T\tilde{\Theta}_2) \\ & -\frac{1}{2}\left[\frac{1}{\rho^2}\underline{\mathrm{e}}^T CC^T\mathrm{e}-2\frac{\underline{\mathrm{e}}^T C}{\rho}\rho(\omega-d)\right. \\ & \left.+\rho^2(\omega-d)^T(\omega-d)\right]. \end{aligned} \quad (60)$$

Substituting adaptation laws (31), (44), and (45) into (60), results in

$$\dot{V} \le -\frac{1}{2}\underline{\hat{e}}^T Q_1'\underline{\hat{e}}-\frac{1}{2}\underline{e}^T Q_2'\underline{e}+\frac{\rho^2}{2}(\omega-d)^T(\omega-d). \quad (61)$$

Integrating the above inequality from $t=0$ to T yields to

$$V(T)-V(0) \le -\frac{1}{2}\int_0^T \underline{\hat{\mathrm{e}}}^T Q_1'\underline{\hat{\mathrm{e}}}\,dt-\frac{1}{2}\int_0^T \underline{\mathrm{e}}^T Q_2'\underline{\mathrm{e}}\,dt + \frac{\rho^2}{2}\int_0^T (\omega-d)^T(\omega-d)\,dt \quad (62)$$

as $V(T)\ge 0$, the following H_∞ criterion is obtained:

$$\frac{1}{2}\int_0^T \underline{\hat{\mathrm{e}}}^T Q_1'\underline{\hat{\mathrm{e}}}\,dt+\frac{1}{2}\int_0^T \underline{\mathrm{e}}^T Q_2'\underline{\mathrm{e}}\,dt \le V(0)+\frac{\rho^2}{2}\int_0^T (\omega-d)^T(\omega-d)\,dt. \quad (63)$$

If $(\omega-d)\in L^2$, then by using Barbalat's lemma [12], it can be proved that the error trajectories of dynamical model (53) will asymptotically converges to the origin, Therefore, the signal $\underline{x}(t)$ of system will track the reference signal $\underline{x}_d(t)$. □

Remark 2 The preceding theorem is also valid when $f_{i1}(\underline{x}(t))$ and $G_i(\underline{x}(t))$ $(i=1,2,\ldots,m)$ in (10) are completely unknown. In this condition, F_{01} and G_0 of (11) will be assumed to be equal to zero and it can be concluded that (27) and (28) will approximate $\Delta F_1(\underline{x}(t))$ and $\Delta G(\underline{x}(t))$ of (11) completely.

Remark 3 To overcome the singularity of

$$[\hat{G}(\underline{\hat{x}}(t)|\Theta_2)+G_0]$$

in (40), different methods such as projection algorithm [8], and exchanging

$$[\hat{G}(\underline{\hat{x}}(t)|\Theta_2)+G_0]^{-1}$$

for its regularized inverse, have been proposed [27].

5 Simulation results

In this section, to illustrate the validity of the proposed IAIT2F controller, tracking problem of a robotic manipulator is simulated in three subsections. In Sect. 5.1 Case 1, the tracking problem of a robotic manipulator with 2-DOF is simulated, and the results are compared with the type-1 fuzzy controller proposed by [24]. In Sect. 5.2, the same control law is proved to be effective while considering time-delayed dynamic of the manipulator and the results the comparisons with the type-1 fuzzy controller of [24] is presented, also. Finally, a backlash-like nonlinearity is added to the control input of the robot in Sect. 5.3 and it will be proved that, the proposed control law is robust to this kind of input nonlinearities and the superiority of our method compared to the proposed method in [24] is illustrated.

All the simulation results are implemented within MATLAB software and with the step size of 0.01.

5.1 Case 1

Consider a robotic manipulator as shown in Fig. 5.

To propose a dynamic model for this robotic manipulator, base on [24], following equations are written:

$$\begin{pmatrix}\ddot{q}_1\\ \ddot{q}_2\end{pmatrix}=\begin{pmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}^{-1}\left\{\begin{pmatrix}u_1\\ u_2\end{pmatrix} -\begin{pmatrix}-h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2)\\ h\dot{q}_1 & 0\end{pmatrix}\begin{pmatrix}\dot{q}_1\\ \dot{q}_2\end{pmatrix}\right\} \quad (64)$$

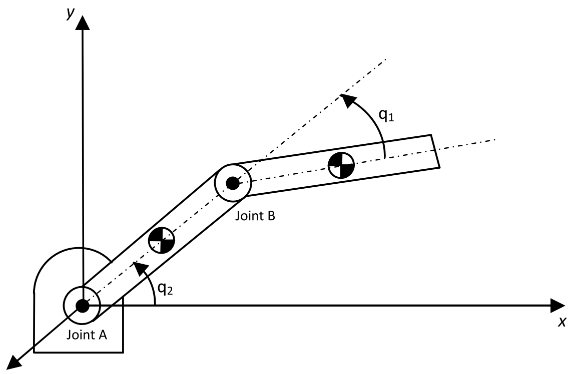

Fig. 5 Robotic Manipulator with two DOF

where

$$M_{11} = a_1 + 2a_3\cos(q_2) + 2a_4\sin(q_2),$$
$$M_{12} = M_{21} = a_2 + a_3\cos(q_2) + a_4\sin(q_2),$$
$$M_{22} = a_2,$$
$$h = a_3\sin(q_2) - a_4\cos(q_2)$$

and

$$a_1 = I_1 + m_1 {l_{c1}}^2 + I_e + m_e {l_{ce}}^2 + m_e {l_1}^2,$$
$$a_2 = I_e + m_e {l_{ce}}^2,$$
$$a_3 = m_e l_1 l_{ce}\cos\delta_e,$$
$$a_4 = m_e l_1 l_{ce}\sin\delta_e.$$

To run the numerical simulation of the manipulator, the constant parameters are chosen as

$$m_1 = 1, \qquad m_e = 2,$$
$$l_1 = 1, \qquad l_{c1} = 0.5, \qquad l_{ce} = 0.6$$
$$I_1 = 0.12, \qquad I_e = 0.25, \qquad \delta_e = \frac{\pi}{6}.$$

Now, to adapt (64) to the general form proposed in (10), consider the notations

$$y = [q_1, q_2]^T, \qquad u = [u_1, u_2]^T,$$
$$x = [\mathrm{q}_1, \dot{\mathrm{q}}_1, \mathrm{q}_2, \dot{\mathrm{q}}_2]^T,$$
$$F(\underline{x}) = \begin{pmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}^{-1} \times \begin{pmatrix} -h\dot{q}_2 & -h(\dot{q}_1 + \dot{q}_2) \\ h\dot{q}_1 & 0 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix}, \tag{65}$$
$$G(\underline{x}) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}^{-1}.$$

Then the state space form of dynamic equation (65) can be written in the form of

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(\underline{x}) + g_{11}(\underline{x})u_1 + g_{12}(\underline{x})u_2 + d_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(\underline{x}) + g_{21}(\underline{x})u_1 + g_{22}(\underline{x})u_2 + d_2 \\ y_1 = x_1 \\ y_2 = x_3. \end{cases} \tag{66}$$

In following tracking simulations, the reference trajectory is defined as $y_m = [\sin(t), \sin(t)]^T$.

To solve the Riccati equation (38) and obtain the robust controller (41), R is computed by $R = 2\rho^2 I$ where $\rho = 0.04$ and adaptation gains are given as $\gamma_0 = 2$, $\gamma_1 = 5$ and $\gamma_2 = 15$.

In order to choose the initial conditions of fuzzy approximators, the interval type-2 fuzzy membership functions are all designed as uncertain Gaussian functions (Fig. 2), with variance $\sigma = 2$. In addition, -0.5 displacement and $+0.5$ displacement from following centers, are chosen for the left and right mean of each membership function, respectively:

$$\theta_{f_1} = \theta_{f_2} = 1_{7\times1},$$
$$\theta_{g_{11}} = [0\ {-7}\ 4\ 1\ 3\ 1\ 5]^T,$$
$$\theta_{g_{12}} = [-3\ 2\ 6\ 0\ 3\ {-3}\ 0]^T,$$
$$\theta_{g_{21}} = [-7\ {-5}\ 0\ {-9}\ 0\ 5\ 1]^T,$$
$$\theta_{g_{22}} = [-6\ 4\ 0\ 6\ 18\ 9\ 7]^T.$$

The observer and control gain matrices are given as

$$K_C = \begin{pmatrix} 0 & 20 \\ 0 & 20 \\ 5 & 0 \\ 5 & 0 \end{pmatrix}, \qquad K_O = \begin{pmatrix} 80 & 0 \\ 800 & 0 \\ 0 & 80 \\ 0 & 800 \end{pmatrix}.$$

Finally, the initial conditions of state equations and observer states are selected as $\underline{x}(0) = [1, 1, 0.5, 0]^T$, $\hat{\underline{x}}(0) = [1, 0, 0.5, 0]^T$, respectively.

To illustrate system output trajectories and control signals of the proposed method, results of proposed controller compared to the controller of [24] are shown in Figs. 6, 7, and 8.

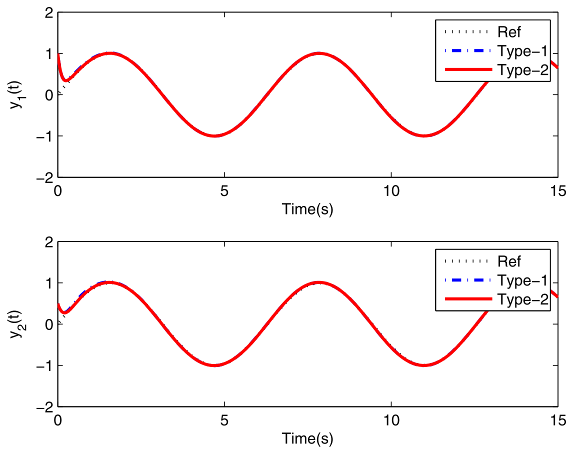

Fig. 6 Output state trajectories (y_1, y_2) tracking the reference signals

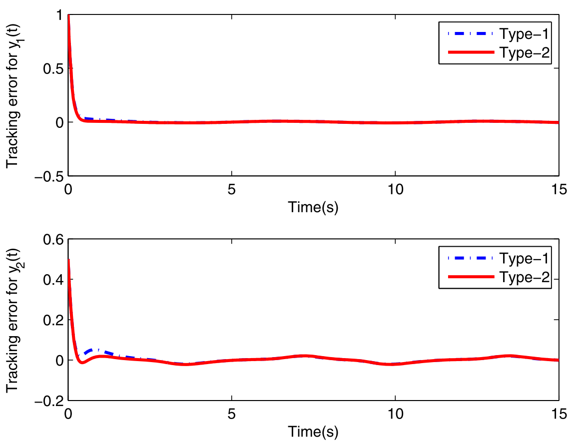

Fig. 7 Tracking errors (e_1, e_2)

Overall, the simulations show that the proposed method performs well and the nonlinear system achieves the desired tracking performance in presence of parameter uncertainties. Comparing the type-1 controller of [24] with our proposed method, both controllers have similar performance, while as shown in Fig. 8, our control scheme needs less control effort in the first second of operation.

Moreover, it should be mentioned that the main advantages of our proposed controller compared to [24] will be illustrated in next two subsections in the presence of time-delays and backlash nonlinearities.

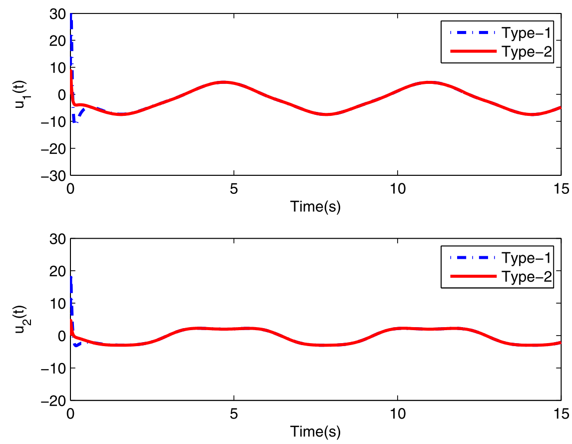

Fig. 8 Control signals (u_1, u_2)

5.2 Case 2

In this subsection, the delayed behavior of manipulator links is considered. Due to inertia effect this delay is unavoidable [33] and it can emerge in different forms, with time-varying property. Here, the time-delay is considered as $0.04\sin(t)$, which is added to the 4th state (i.e., velocity of 2nd joint) of (66). Therefore, (66) can be rewritten in the form of

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(\underline{x}(t)) + g_{11}(\underline{x}(t))u_1 + g_{12}(\underline{x}(t))u_2 + d_1 \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f_2(\underline{x}(t - 0.04\sin(t))) + g_{21}(\underline{x}(t))u_1 \\ \qquad + g_{22}(\underline{x}(t))u_2 + d_2 \\ y_1(t) = x_1(t) \\ y_2(t) = x_3(t). \end{cases} \tag{67}$$

In this subsection, all other parameters of control laws are the same as Sect. 5.1. States trajectories, tracking error signals and control inputs are shown in Figs. 9, 10, and 11.

Based on Fig. 10, it can be seen that although the proposed IAIT2F controller still retain its control performance, unpredicted tracking errors emerge in the type-1 control scheme of [24]. In addition, Fig. 11 illustrates that the proposed type-2 method needs less control effort comparing with the type-1 method of [24].

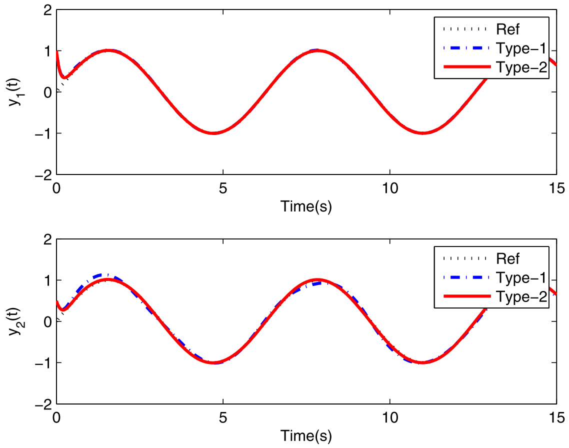

Fig. 9 Output state trajectories (y_1, y_2) tracking the reference signals

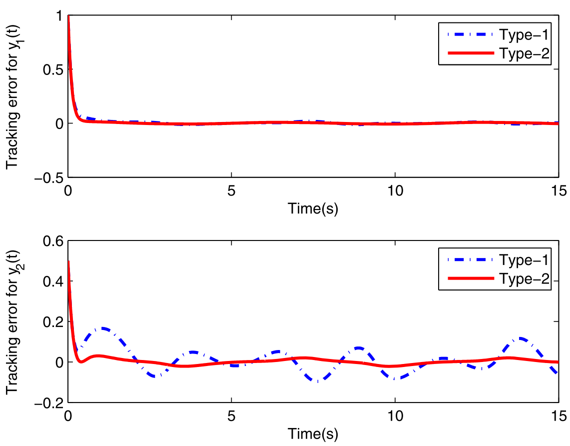

Fig. 10 Tracking errors (e_1, e_2)

5.3 Case 3

Based on Theorem 1, it is proved that even in presence of hysteresis nonlinearities, system tracking performance is guaranteed. Therefore, in this subsection a backlash-like hysteresis as shown in Fig. 1 is added to the control input. The parameters used in (4) are as $\alpha = 0.3$, $c = 1$, and $B_1 = 1$.

To show the high robustness of proposed IAIT2F controller compared to the type-1 controller of [24], except including the backlash-like hysteresis, all other simulation conditions are the same as in Sect. 5.2. Figures 12, 13, and 14 illustrate the results of comparisons.

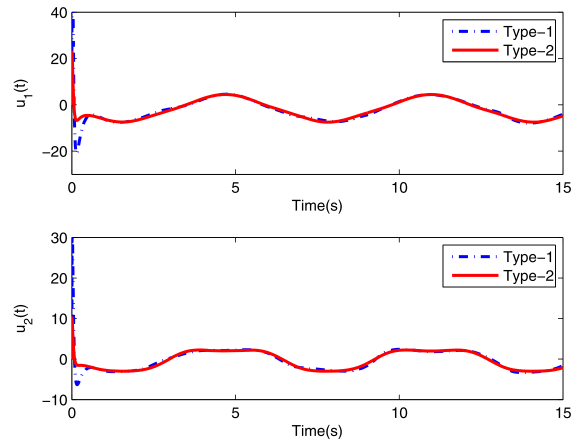

Fig. 11 Control signals (u_1, u_2)

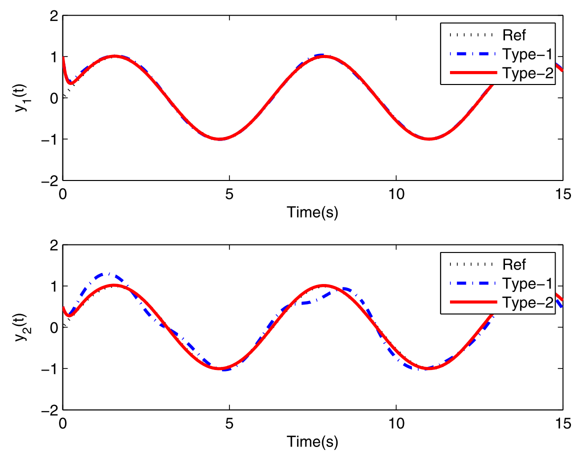

Fig. 12 Output state trajectories (y_1, y_2) tracking the reference signals

As can be seen in Figs. 12 and 13, the type-1 method of [24] does not support the backlash-like hysteresis property of control input and it concludes to undesirable tracking errors. However, the proposed type-2 method is completely robust stable in presence of backlash-like nonlinearities and it can maintain system robust stability with much less control effort.

5.4 Quantitative comparisons

In order to present a quantitative comparison of control efforts and control performance among the simulation results, Integral of Absolute Error (IAE) and Integral of Absolute $u(t)$ (IAU) are used as the criteria and the results of aforementioned three simulation case stud-

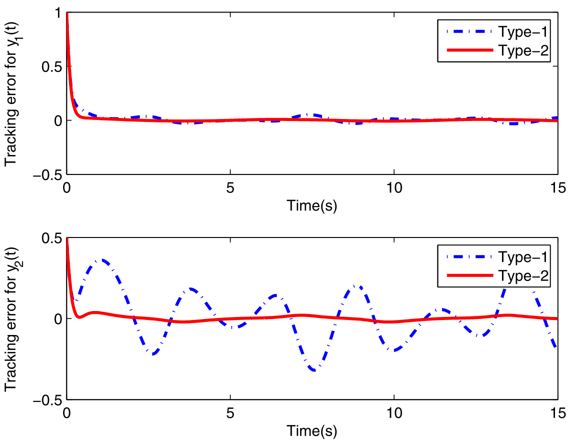

Fig. 13 Tracking errors (e_1, e_2)

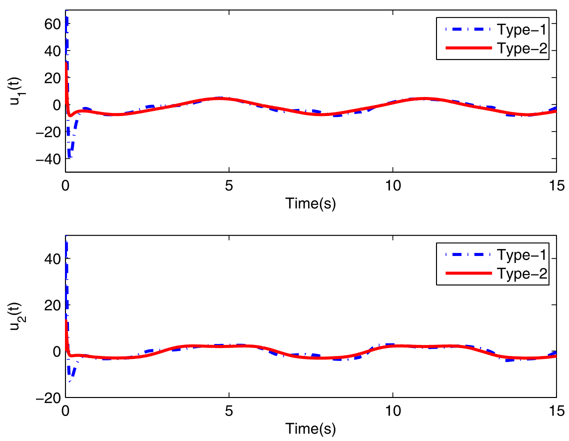

Fig. 14 Control signals (u_1, u_2)

Table 1 Quantitative comparisons of proposed IAT2F controller with the type-1 method of [24]

		IAE_1	IAE_2	IAU_1	IAU_2
Case 1	(Type-1)	0.17	0.19	63.7	31.7
	(Type-2)	0.14	0.16	61	30
Case 2	(Type-1)	0.22	0.76	66.7	32.7
	(Type-2)	0.15	0.18	62.0	30.5
Case 3	(Type-1)	0.38	1.99	71.9	35.9
	(Type-2)	0.17	0.21	62.6	30.8

ies for both type-1 method of [24] and our proposed IAT2F control scheme are presented in Table 1.

The numerical comparisons of Table 1 shows that the proposed type-2 controller is robust to time-delayed dynamic and input nonlinearities, while the type-1 method of [24] cannot maintain its stability in case of input nonlinearities and time varying time-delays.

6 Conclusion

We presented an approach for controlling a class of MIMO nonlinear systems containing input hysteresis nonlinearity, time varying time-delays, model uncertainties, and external disturbances. The proposed method is based on H^∞ robust control technique and fuzzy logic systems. The control input comprises an adaptive interval type-2 fuzzy system which approximates the uncertain model, and an H^∞-based controller, which attenuates the effects of external disturbances and approximation errors to a prescribed level. Moreover, a state observer was used to estimate the unknown states. Based on Lyapunov theory, we proved the stability of the closed-loop system to ensure that desired robustness always occur. Finally, the effectiveness of the designed controller was illustrated through the simulations and results were compared with an existing method.

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