A Robust H^∞ Control Design for Swarm Formation Control of Multi-Agent Systems: A Decentralized Adaptive Fuzzy Approach

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***Abstract*—In this paper, a decentralized adaptive control scheme for multi-agent formation control is proposed. This control method is based on artificial potential functions integrated with adaptive fuzzy H^∞ technique. We consider fully actuated mobile agents with partially unknown models, where an adaptive fuzzy logic system is used to approximate the unknown system dynamics. The H^∞ control theory is used to attenuate the adaptive fuzzy approximation error to a prescribed level. Therefore the agents motion is forced to obey the dynamics defined by the simple inter-agent artificial potential functions. Stability proof is given using Lyapunov functions, which shows the robust behavior of controller with respect to disturbances and system uncertainties. Finally, simulation results are demonstrated for a multi-agent formation problem of a group of six agents, illustrating the effective attenuation of fuzzy logic approximation error.**

***Index Terms*—multi-agent systems, formation control, H$^\infty$ control theory, adaptive fuzzy logic systems.**

I. Introduction

Multi-agent systems are very interesting decentralized systems and have been studied extensively over the past years [1]–[5]. These systems have the complex behavior usually seen in large-scale systems, although each agent is associated with simple dynamics. Therefore, the decentralized control of multi-agent systems have received increased research attention [6]–[8], since the pioneering work on birds flocking behavior by Reynolds [1] in 1987.

Applications of multi-agent systems are in different fields of industrial and martial applications, such as underwater or outer space exploration, hazardous inspiration and guarding, escorting and patrolling missions [2], [9], [10]. It must be mentioned that in a wider sense, multi-agent systems can play an important role in resolving outstanding problems in network communications [7].

Many techniques for multi-agent systems control have been developed in recent years. These include potential fields [2], behavior-based [5], leader-following [4], graph-theoretic [11] and virtual structure approaches [3], [12]. All of these techniques are categorized as three main types of the control task allocation, which are *Centralized*, *Decentralized* and *Hybrid* control. A small group of robots can be controlled by a central computer using centralized approach [13]. However, limitations in computational power and communication bandwidth, limit the number of robots. Therefore decentralized and hybrid controllers are interested for a large group of robots [11].

Almost all of the above techniques have been used to address the formation control problem. This problem is defined as the organization of a swarm of agents into a particular shape in a $2D$ or $3D$ space [2]. Biological systems provide good examples on such problem, e.g. an ant colony utilizes some behavior-based rules for each ant to achieve a special colony shape.

Most of the formation control methodologies cited before either consider a group of mass-less agents with kinematic model [2] or consider known dynamic models for each agent. However, practical multi-agent systems inherently contain nonlinearities and system uncertainties which are commonly unknown to the system designer. Therefore, in modeling and analysis of such large-scale systems, one needs to handle unknown nonlinearities and/or uncertain parameters. On the other hand, measurement noises may be included as some known disturbances to the original signal. Thus, many adaptive and robust control techniques has been proposed, which consider the existence of disturbances in the multi-agent system [3], [12], [14].

Since Zadeh [15] initiated the fuzzy set theory, Fuzzy Logic Systems (FLS) have been widely applied to many real world applications. Besides, FLS schemes have been widely used in motion control of single robots [16], [17]. Using FLS integrated with H^∞ control technique can improve the robustness of controller and ensures the stability [18].

In an H^∞ control technique, the main design goal is to force the gain from unmodelled dynamics, external disturbances and approximation errors to be equal or less than a prescribed disturbance attenuation level (H^∞ attenuation constraint) [18]–[20]. This goal is generally represented as a Linear Matrix Inequality (LMI) problem.

The main contribution of this paper is to propose a simple adaptive control scheme to tackle the mentioned limitations. Besides, the performance of the proposed control technique which is based on fuzzy logic systems has been also improved using H$^\infty$ technique.

Using the Lyapunov theory, the stability of the proposed adaptive controller is proved. It can be seen that the proposed control method can be applied to a wide class of uncertain

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multi-agent systems but also it is simple to implement in practical applications. Simulation results indicate the effectiveness of the proposed method.

The rest of this paper is organized as follows: Section II presents the system description and problem formulation. Design of the proposed controller and stability analysis are discussed in Section III and Section IV, respectively. Simulation results are included in Section V. Section VI provides the concluding remarks.

II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

The major goal in a multi-agent formation control problem is to control the relative position and orientation of the agents to create a desirable formation. In Subsection II-A we propose a potential function design for formation control of a group of N point mass-less agents. The kinematic of the i^{th} robot is written as

$$\dot{z}_i = u_i \quad i \in \{1, 2, ..., n\}, \tag{1}$$

where $z_i \in \mathbb{R}^n$ is the coordinate matrix (for a robot with n-degrees of freedom) and $u_i \in \mathbb{R}^n$ denotes the control inputs. However, one of the main shortcomings of this kinematic model is that it does not correspond to the dynamics of realistic agents. Therefore, in Subsection II-B a general n-degrees of freedom dynamic model of real robots is considered to propose more realistic solutions for formation control of multi-agent systems.

A. Formation control of agents without considering mass of agents

To propose a control law, an artificial potential function is considered and pair-wise potential fields are defined between agents as

$$F_{ij} = L_{ij}\left(|z_i - z_j|\right), \forall i, j \in \{1, 2, ..., n\}, \tag{2}$$

where L_{ij} is designed to define a proper inter-agent potential function. It is assumed that each agent senses the resultant potential of all other agents.

The overall potential function is proposed to be in the form of

$$F = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} L_{ij}\left(|z_i - z_j|\right) + \sum_{i=1}^{N} Q_i\left(|z_i|\right), \tag{3}$$

where Q_i defines the global potential of each agent.

Finally, the following three assumptions for potential function in (3) is considered [3], [12]:

Assumption 1: F is Continuously differentiable.

Assumption 2: F is Strictly convex.

Assumption 3: F is Positive definite.

For example, the following potential function can be chosen for a desired polygonal formation in a $2D$ Space:

$$F = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(|z_i - z_j|^2 - d_{ij}\right)^2 + \sum_{i=1}^{N} \left(|z_i|^2 - r_i\right)^2 \tag{4}$$

at the first step, to propose a solution for multi-agent formation control, the steepest descent direction ([2], [3], [12]) is chosen as

$$f_i = \frac{\partial F}{\partial z_i} \tag{5}$$

and the control law

$$u_i = -f_i, \forall i \in \{1, 2, ..., n\} \tag{6}$$

is proposed.

By substituting (6) in (1) the kinematic model is obtained as

$$\dot{z}_i = -f_i = -\frac{\partial F}{\partial z_i}, \forall i \in \{1, 2, ..., n\} \tag{7}$$

which can be rewritten in the matrix form as $\dot{Z} = -\nabla F$ where $Z = [z_1, z_2, ..., z_n]$ is the overall generalized coordinate vector.

In the next subsection, it is proposed to assume the multi-agent system with a general dynamic model. Furthermore, in Section III a robust adaptive fuzzy controller using an H^∞ approach is used to force the satisfaction of (7). In other words the proposed controller is designed to enforce the speed of each agent along the negative gradient of potential function in (3).

B. Formation control of robots with Dynamic models

In this subsection a general dynamic model [21] is addressed to represent any kind of autonomous n degrees-of-freedom system. This model has been previously used in some existing works (e.g. [3], [12]).

Consider a group of N fully autonomous robots. The dynamics of the i^{th} simple robot is strongly nonlinear [21] and can be written in the general form

$$M(z_i)\ddot{z}_i + C(z_i, \dot{z}_i)\dot{z}_i + g(z_i) = u_i, \tag{8}$$

where $z_i \in \mathbb{R}^n$ is the coordinate matrix (for a robot with n-degrees of freedom), $M(z_i) \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix and represents the inertia coefficients. $C(z_i, \dot{z}_i) \in \mathbb{R}^{n \times n}$ is the matrix of centripetal, Coriolis, damping and rolling resistance forces, $g(z_i) \in \mathbb{R}^n$ is an n-vector of gravitational forces and $u_i \in \mathbb{R}^n$ denotes the control inputs.

In most practical control problems of multi-agent systems the inertia matrix $M(z_i)$ is a known constant matrix independent of z_i. Therefore, the following assumption is considered:

Assumption 4: M is the inertia matrix of robots, which is assumed to be a known and constant matrix.

Let us rewrite (8) as

$$M\ddot{z}_i + C(z_i, \dot{z}_i)\dot{z}_i + g(z_i) = u_i. \tag{9}$$

It is straightforward to rewrite (9) as

$$\ddot{z}_i = -M^{-1}C(z_i, \dot{z}_i)\dot{z}_i - M^{-1}g(z_i) + M^{-1}u_i. \tag{10}$$

In the next section, the dynamic of each single robot will be assumed to be in the form of (10).

III. Controller Design Methodology

In this section a novel formation error based on the integral of formation gradient (5) will be proposed. Then, a robust H^{∞} controller will be designed and a fuzzy logic system will be utilized to approximate the unknown parts of dynamic models. The main feature of the proposed novel control scheme is its decentralized characteristic, robustness to external disturbances, input nonlinearities and measurement noises. Besides, by using the proposed controller, the formation can be achieved from any initial conditions.

Consider, the novel formation error for the i^{th} robot as

$$\underline{e}_i(t) = z_i(t) + \int_0^t f_i(\tau)d\tau \tag{11}$$

where $\underline{e_i} \in \mathbb{R}^n$, z_i represents the coordinate vector of i^{th} robot in (8) and f_i is the gradient of potential function defined in (5). It is straightforward to write the first and second derivatives of (11) as

$$\dot{\underline{e}}_i(t) = \dot{z}_i(t) + f_i \tag{12}$$

and

$$\ddot{\underline{e}}_i(t) = \ddot{z}_i(t) + \dot{f}_i. \tag{13}$$

Our design goal is to propose an adaptive fuzzy controller so that

$$\ddot{\underline{e}}_i + k_1\dot{\underline{e}}_i + k_2\underline{e}_i = 0 \tag{14}$$

is achieved, where k_1 and k_2 are chosen to make (14) asymptotically stable.

To design the controller, control law is proposed as

$$u_i = M\big(H_i(z_i, \dot{z}_i) - \dot{f}_i - k_1\dot{\underline{e}}_i - k_2\underline{e}_i\big) \tag{15}$$

where

$$H_i(z_i, \dot{z}_i) = M^{-1}C_i(z_i, \dot{z}_i)\dot{z}_i + M^{-1}g(z_i). \tag{16}$$

In order to use this control law, the function $H_i(.)$ (i.e. $g(.)$ and $C(.)$) must be known. However, in practice these matrices may be unknown for most of real dynamical robots. To overcome this, we make use of an adaptive fuzzy logic system $\hat{H}_i(.)$ to approximate $H_i(.)$.

Therefore, using the singleton fuzzifier, product inference, and weighted average defuzzifier [22], the output of the fuzzy model can be expressed as

$$\hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) = \underline{\zeta}_i^T(z_i, \dot{z}_i)\underline{\theta}_i, \tag{17}$$

where

$$\underline{\zeta}_i = \begin{bmatrix} \underline{\zeta}_{1i}^T & 0 & \dots & 0 \\ 0 & \underline{\zeta}_{2i}^T & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \underline{\zeta}_{ni}^T \end{bmatrix}, \underline{\theta}_i = \begin{bmatrix} \underline{\theta}_{1i} \\ \underline{\theta}_{2i} \\ \vdots \\ \underline{\theta}_{ni} \end{bmatrix}$$

Eq. (17) suggests us to rewrite the overall control law (15) as

$$u_i = M\big(\hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) - \dot{f}_i - k^T\underline{e}_i - \underline{u}_{ai}\big) \tag{18}$$

where u_{ai} is engaged to attenuate the fuzzy logic approximation error.

It should be mentioned that, the issue of agent collision is not addressed directly in the proposed method. However, some small modifications on the artificial potential functions can handle this problem. The terms defined in (3) are known as attraction functions, and including inter-agent repulsion potentials as discussed in [2] can easily lead to the collision avoidance.

A block diagram of the proposed control methodology is shown in Fig. 1.

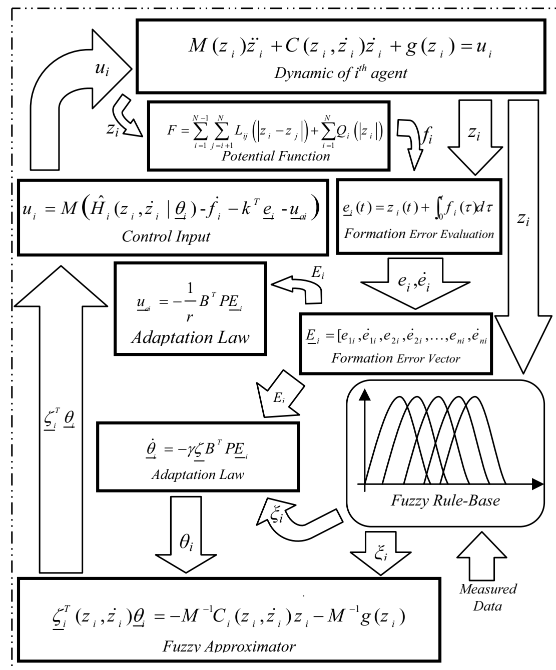

Fig. 1. Block diagram of the proposed adaptive fuzzy H^{∞} control scheme

IV. Stability Analysis

To derive the adaptive law for adjusting $\underline{\theta}_i$, we first define the optimal parameter vector $\underline{\theta}_i^*$ as

$$\underline{\theta}_i^* = \arg\min_{\underline{\theta}_i \in \Omega} \left[\sup \left\|\hat{H}_i(z, \dot{z}|\theta_i) - H_i(z, \dot{z})\right\|\right] \tag{19}$$

and the minimum approximation error is defined as

$$\underline{w}_i = H_i(z_i, \dot{z}_i) - \hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i^*), \tag{20}$$

where it can be assumed that $\underline{w}_i \in L_\infty$ [22].

By choosing the control input as (18) after some manipulations, (10) can be rewritten as

$$\ddot{z} + \dot{f}_i = \Big(\hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) - H_i(z_i, \dot{z}_i)\Big) + k_1\dot{\underline{e}}_i + k_2\underline{e}_i - \underline{u}_{ai} \tag{21}$$

and the formation error dynamic can be expressed as

$$\ddot{\underline{e}}_i = \Big(\hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i) - H_i(z_i, \dot{z}_i)\Big) + k_1\dot{\underline{e}}_i + k_2\underline{e}_i - \underline{u}_{ai}. \tag{22}$$

Moreover by defining $\underline{E}_i = [e_{1i}, \dot{e}_{1i}, e_{2i}, \dot{e}_{2i}, \dots, e_{ni}, \dot{e}_{ni}]$ it is straightforward to write

$$\dot{\underline{E}}_i = A\underline{E}_i + B\underline{u}_{ai} + B(H_i(z_i, \dot{z}_i) - \hat{H}_i(z_i, \dot{z}_i|\underline{\theta}_i)) \tag{23}$$

where

$$A = I_{n\times n} \otimes \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}_{2\times 2}, \quad B = I_{n\times n} \otimes [0 \;\; 1]^T.$$

Based on (17), (19) and (20), the matrix form of formation error in (23) can be rewritten as

$$\dot{\underline{E}}_i = A\underline{E}_i + B\underline{u}_{ai} + B\underline{\zeta}_i^T(z_i, \dot{z}_i)\tilde{\underline{\theta}}_i + B\underline{w}_i, \tag{24}$$

where $\tilde{\underline{\theta}}_i = \underline{\theta}_i - \underline{\theta}_i^*$.

In the following theorem, it will be shown that, the proposed control law (18) guarantees the stability and robustness of formation problem.

Theorem 1: Consider a group of N fully autonomous robots with the dynamic represented in (9), and with the control law in (18). The robust compensator of i^{th} robot $\underline{u}_{ai}$ and the fuzzy adaptation law are chosen as

$$\underline{u}_{ai} = -\frac{1}{r}B^T P\underline{E}_i \tag{25}$$

and

$$\dot{\underline{\theta}}_i = -\gamma\underline{\zeta}(z_i, \dot{z}_i)B^T P\underline{E}_i \tag{26}$$

where r and γ are positive constants and P is the positive semidefinite solution of following Riccati-like equation:

$$PA + A^T P + Q - \frac{2}{r}PBB^T P + \frac{1}{\rho^2}PBB^T P = 0 \tag{27}$$

where Q is a positive semidefinite matrix and $2\rho^2 \geq r$.

Therefore, the H^∞ tracking performance

$$\begin{aligned} &\textstyle\sum_{i=1}^N \left[-\int_0^T \underline{E}_i^T Q\underline{E}_i \; dt\right] \\ &\quad \leq \textstyle\sum_{i=1}^N \left[\underline{E}_i(0)^T P\underline{E}_i(0) + \frac{1}{\gamma}\tilde{\underline{\theta}}(0)_i^T\tilde{\underline{\theta}}_i(0)\right] \\ &\qquad + \textstyle\sum_{i=1}^N \left[\rho^2 \int_0^T \underline{w}_i^T\underline{w}_i \; dt\right] \end{aligned} \tag{28}$$

can be achieved for a prescribed attenuation level ρ and all the variables of closed loop system are bounded.

Proof 1: In order to derive the adaptive law for adjusting $\underline{\theta}_i$, the Lyapunov candidate is chosen as

$$V = \sum_{i=1}^N \left[\frac{1}{2}\underline{E}_i^T P\underline{E}_i + \frac{1}{2\gamma}\tilde{\underline{\theta}}_i^T\tilde{\underline{\theta}}_i\right]. \tag{29}$$

Using (24), the time derivative of V is

$$\begin{aligned} \dot{V} &= \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\dot{\underline{E}}_i^T P\underline{E}_i + \underline{E}_i^T P\dot{\underline{E}}_i + \frac{1}{\gamma}\dot{\tilde{\underline{\theta}}}_i^T\tilde{\underline{\theta}}_i + \frac{1}{\gamma}\tilde{\underline{\theta}}_i^T\dot{\tilde{\underline{\theta}}}_i\Big] \\ &= \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\underline{E}_i^T A^T P\underline{E}_i + \underline{u}_{ai}^T B^T P\underline{E}_i \\ &\quad + \tilde{\underline{\theta}}_i^T\underline{\zeta}_i(z_i, \dot{z}_i)B^T P\underline{E}_i + \underline{w}_i^T B^T P\underline{E}_i + \underline{E}_i^T PA\underline{E}_i \\ &\quad + \underline{E}_i^T PB\underline{u}_{ai} + \underline{E}_i^T PB\underline{\zeta}_i^T(z_i, \dot{z}_i)\tilde{\underline{\theta}}_i + \underline{E}_i^T PB\underline{w}_i\Big] \\ &\quad + \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\frac{1}{\gamma}\dot{\tilde{\underline{\theta}}}_i^T\tilde{\underline{\theta}}_i + \frac{1}{\gamma}\tilde{\underline{\theta}}_i^T\dot{\tilde{\underline{\theta}}}_i\Big]. \end{aligned} \tag{30}$$

Substituting (25) in (30) and using the fact that $\dot{\tilde{\underline{\theta}}}_i = \dot{\underline{\theta}}_i$, we get

$$\begin{aligned} \dot{V} &= \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\underline{E}_i^T A^T P\underline{E}_i - \frac{1}{r}\underline{E}_i^T PBB^T P\underline{E}_i \\ &\quad + \tilde{\underline{\theta}}_i^T\underline{\zeta}_i(z_i, \dot{z}_i)B^T P\underline{E}_i + \underline{w}_i^T B^T P\underline{E}_i \\ &\quad + \underline{E}_i^T PA\underline{E}_i - \frac{1}{r}\underline{E}_i^T PBB^T P\underline{E}_i \\ &\quad + \underline{E}_i^T PB\underline{\zeta}_i^T(z_i, \dot{z}_i)\tilde{\underline{\theta}}_i + \underline{E}_i^T PB\underline{w}_i\Big] \\ &\quad + \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\frac{1}{\gamma}\dot{\underline{\theta}}_i^T\tilde{\underline{\theta}}_i + \frac{1}{\gamma}\tilde{\underline{\theta}}_i^T\dot{\underline{\theta}}_i\Big] \\ &= \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\underline{E}_i^T\Big(A^T P + PA - \frac{2}{r}\underline{P}BB^T P\Big)\underline{E}_i\Big] \\ &\quad + \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\Big(\underline{E}_i^T PB\underline{\zeta}_i^T(z_i, \dot{z}_i) + \frac{1}{\gamma}\dot{\underline{\theta}}_i^T\Big)\tilde{\underline{\theta}}_i\Big] \\ &\quad + \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\underline{w}_i^T B^T P\underline{E}_i + \underline{E}_i^T PB\underline{w}_i\Big]. \end{aligned} \tag{31}$$

Using adaptation law (26) and the Riccati-like equation (27), the above equation becomes

$$\begin{aligned} \dot{V} &= \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[-\underline{E}_i^T Q\underline{E}_i - \frac{1}{\rho^2}\underline{E}_i^T PBB^T P\underline{E}_i\Big] \\ &\quad + \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[\underline{w}_i^T B^T P\underline{E}_i + \underline{E}_i^T PB\underline{w}_i\Big] \\ &= \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[-\underline{E}_i^T Q\underline{E}_i \\ &\quad - \big(\frac{1}{\rho}B^T P\underline{E}_i - \rho\underline{w}_i\big)^T\big(\frac{1}{\rho}B^T P\underline{E}_i - \rho\underline{w}_i\big)\Big] \\ &\quad + \tfrac{1}{2}\textstyle\sum_{i=1}^N \big[\rho^2\underline{w}_i^T\underline{w}_i\big] \\ &\leq \tfrac{1}{2}\textstyle\sum_{i=1}^N \Big[-\underline{E}_i^T Q\underline{E}_i + \rho^2\underline{w}_i^T\underline{w}_i\Big]. \end{aligned} \tag{32}$$

Integrating the above inequality from $t = 0$ to T yields to

$$V(T) - V(0) \leq \frac{1}{2}\sum_{i=1}^{N}\left[-\int_0^T \underline{E}_i^T Q \underline{E}_i \, dt + \rho^2 \int_0^T \underline{w}_i^T \underline{w}_i \, dt\right]. \tag{33}$$

Using the fact that $V(T) \geq 0$ and from (29), the inequality

$$\frac{1}{2}\sum_{i=1}^{N}\left[-\int_0^T \underline{E}_i^T Q \underline{E}_i \, dt\right] \leq \frac{1}{2}\sum_{i=1}^{N}\left[\underline{E}_i(0)^T P \underline{E}_i(0) + \frac{1}{\gamma}\underline{\tilde{\theta}}(0)_i^T \underline{\tilde{\theta}}_i(0)\right] + \frac{1}{2}\sum_{i=1}^{N}\left[\rho^2 \int_0^T \underline{w}_i^T \underline{w}_i \, dt\right] \tag{34}$$

is obtained.

Therefore, the H^∞ tracking equation (28) can be achieved and the proof is completed.

V. Simulation Results

This section presents three simulation examples to illustrate the effectiveness of the proposed control scheme. In the first example, we give the simulation results of a simple formation control strategy for a group of six point mass-less agents with the simple kinematic model as (1). In the second example, in the case of six agents with known dynamics the performance of the ideal controller proposed in (15) is investigated. Finally, the third example proves the effectiveness of the proposed adaptive fuzzy H^∞ controller in case of formation control of six agents with partially unknown dynamics.

The unique hexagonal formation problem used in all three simulation examples, is defined by

$$F = \sum_{i=1}^{5}\sum_{j=i+1}^{6}\left(|z_i - z_j|^2 - d_{ij}\right)^2 \tag{35}$$

where $r_i = 1$ is the formation radius and d_{ij} is specified in Table I.

TABLE I
Parameter Specifications of Hexagonal Formation

	$\|i-j\|=1$	$\|i-j\|=2$	$\|i-j\|=3$	$\|i-j\|=4$	$\|i-j\|=5$
d_{ij}	1.0	1.7	2.0	1.7	1.0

A. Point Mass-less Kinematic agent

Consider six agents with the simplest kinematic model (1), the control law (6) and the hexagonal formation defined in (35). The initial positions of each robot is specified in Table II.

Using the gradient method proposed in (6) the agents motion trajectories are shown in Fig. 2.

In this simulation example, the steepest descent based approach introduced in Subsection II-A is proved to be effective.

TABLE II
Agents Initial Positions

Agent No:	*1*	*2*	*3*	*4*	*5*	*6*
x_0	-2.5	+2.0	-1.0	+1.0	+2.0	+2.5
y_0	+1.0	-2.5	+1.0	-1.0	+2.5	-1.0

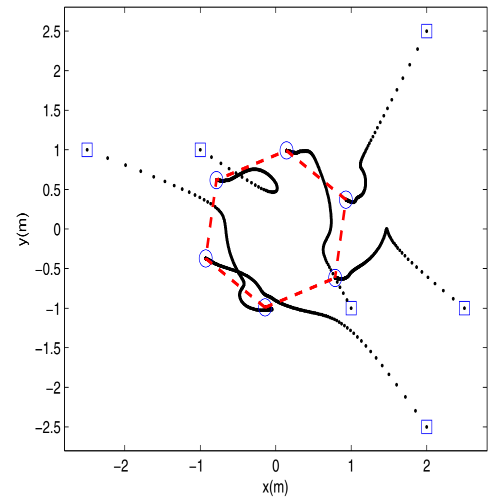

Fig. 2. Hexagonal formation of point mass agents. Square marks show the initial positions of agents and Circle marks show the final positions.

B. Point Mass agents with known dynamic model

Consider a group of six mobile robots with known dynamic models. Based on general model represented in (9), the nonlinear dynamic of the i^{th} robot is considered as

$$\begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix}\begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} + \begin{bmatrix} 0.15 & 0 \\ 0 & 0.15 \end{bmatrix}\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} + \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}\begin{bmatrix} sgn(\dot{x}_i) \\ sgn(\dot{y}_i) \end{bmatrix} = u_i, \tag{36}$$

and after some manipulations we get

$$\begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} = -\begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} - \begin{bmatrix} 0.33 & 0 \\ 0 & 0.33 \end{bmatrix}\begin{bmatrix} sgn(\dot{x}_i) \\ sgn(\dot{y}_i) \end{bmatrix} + \begin{bmatrix} 1.66 & 0 \\ 0 & 1.66 \end{bmatrix} u_i. \tag{37}$$

To give a solution for the formation problem (35), formation error is defined as (11) and the designed control law is proposed based on (15), where $k_1 = 15$ and $k_2 = 4$. Formation trajectory is shown in Fig. 3.

C. Point Mass agent with partially unknown dynamics

To verify the effectiveness of proposed method, we present simulation results for a group of six agents with the same dynamic models as (37).

The formation potential and formation error are chosen as (35) and (11); respectively. However to design the control law, the dynamic model of agents is assumed to be partially unknown. Therefore, six fuzzy logic approximators are designed to approximate the unknown dynamic, where each agent approximator just needs the current position and velocity of itself. Three Gaussian membership functions with unit

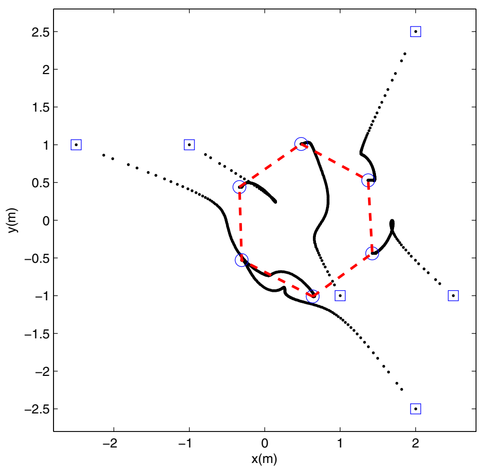

Fig. 3. Hexagonal formation of agents with known dynamic model. Square marks show the initial positions of agents and Circle marks show the final positions.

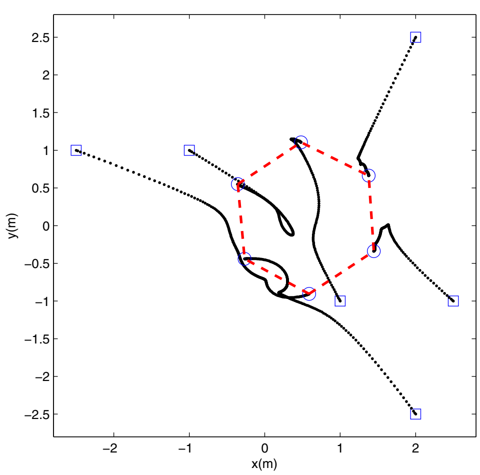

Fig. 4. Hexagonal formation of agents with unknown dynamic model. Square marks show the initial positions of agents and Circle marks show the final positions

variance are defined and all $\underline{\theta}$s are initialized from zero vectors. The learning rate in (26) is set to $\gamma = 15$ and the output of the fuzzy system is achieved by choosing singleton fuzzification, center average defuzzification, Mamdani implication in the rule base and product inference engine [22].

Simulation results of the proposed adaptive fuzzy H^∞ technique with agents initial positions as shown in Table II is shown in Fig. 4.

In this section the effectiveness of the proposed H^∞-based scheme as a solution for formation control problem of multi-agent systems was shown by three simulation examples. In further researches, the proposed methodology will be experimentally examined on a real test-bed of six real mobile robots.

VI. Conclusions

In this paper, a novel formation control strategy for a class of multi-agent systems with partially unknown dynamics was investigated. On the basis of the Lyapunov stability theory, the new H^∞ adaptive controller with corresponding parameter update law was developed. All the theoretical results were verified by simulation examples to demonstrate the effectiveness of the proposed decentralized control scheme.

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