Proceedings of the ASME 2010 10th Biennial Conference on Engineering Systems Design and Analysis
ESDA2010
July 12-14, 2010, Istanbul, Turkey

ESDA2010-25371

ADAPTIVE FUZZY SLIDING MODE CONTROL APPROACH FOR SWARM FORMATION CONTROL OF MULTI-AGENT SYSTEMS

Bijan Ranjbar Sahraei
School of Electrical and Computer Eng.
Shiraz University, Shiraz, Iran

Alireza Nemati
School of Mechanical Eng.
Sharif Uni. of Technology, Tehran, Iran

Mehdi Farshchi
School of Mechanical Eng.
Sharif Uni. of Technology, Tehran, Iran

Ali Meghdari (*ASME Fellow*)
School of Mechanical Eng.
Sharif Uni. of Technology, Tehran, Iran

ABSTRACT
In this paper, an adaptive control scheme for multi-agent formation control is proposed. This control method is based on artificial potential functions integrated with adaptive fuzzy sliding mode control technique. We consider fully actuated mobile agents with completely unknown dynamics. An adaptive fuzzy logic system is used to approximate the unknown system dynamics. Sliding Mode Control (SMC) theory is used to force agents' motion to obey the dynamics defined by the simple inter-agent artificial potential functions. Stability proof is given using Lyapunov functions, which shows the robustness of controller with respect to disturbances and system uncertainties. Simulation results are demonstrated for a multi-agent formation problem, illustrating the effectiveness of the proposed method. Experimental results are included to verify the applicability of the scheme for a test-bed of six real mobile robots.

1. INTRODUCTION
The early works on robots motion control has considered motion of single robots [1-3]. However, in the recent years, control of a multi-agent system consisting swarms of robots has been interested by control community. One of the main reasons for such an interest is to meet the requirement of multi-agent systems in the industrial and military applications. Some other reasons are probably the enormous decrement of the cost of single robots or the emerging of new technologies, which are capable of making compact robots. Some possible applications of a multi-agent system include underwater or outer space exploration, hazardous inspiration and guarding, escorting, patrolling missions [4-6].

In general, a multi-agent formation problem is defined as the organization of a swarm of agents into a particular shape in a 2D or 3D space[4]. Biological systems provide good examples on such problem, e.g. an ant colony utilizes some behavior-based rules for each ant to achieve a special colony shape.

In a formation control problem three main types of the control task allocation are 'centralized', 'decentralized' and 'hybrid' control. A small group of robots can be controlled by a central computer using centralized approach[7]. However, limitations in computational power and communication bandwidth, limit the number of robots. Therefore decentralized and hybrid controller are interested for large number of robots [8].

Several formation control strategies can be found as Potential fields [4], behavior-based [9], leader-following [10], graph-theoretic [8] and virtual structure approaches [11, 12].

In recent years, some methods based on potential fields are integrated with some nonlinear control methods (e.g. Sliding Mode Control (SMC)), which concludes in more robust formation control of dynamic robots [11, 12].

Since Zadeh [13] initiated the fuzzy set theory, Fuzzy Logic Systems (FLS) have been widely applied to many real world applications [14]. Besides, FLS schemes have been widely used in motion control of single robots [15, 16]. Using FLS integrated with SMC can improve the performance of controller and ensures the stability.

In this research, an adaptive fuzzy approximator is combined with SMC to propose a novel adaptive fuzzy formation control methodology. The main advantage of this control strategy is insensitivity to robot dynamic uncertainty and external disturbances.

The rest of this paper is organized as follows: Section 2 presents the system description and problem formulation. Design of the proposed controller and stability proofs are discussed in Section 3 and Section 4, respectively. Simulation results and experimental verification are included in Section 5 and Section 6. Finally, Section 7 provides the concluding remarks.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

The major goal in a multi-agent formation control problem is to control the relative position and orientation of the agents to create a desirable formation. In *Subsection 'A'* we propose a potential function design for formation control of a group of N point mass-less agents. The kinematic of the i^{th} robot is written as

$$\dot{z}_i = u_i, \forall i \in \{1,2,...,n\} \tag{1}$$

where $z_i \in \Re^n$ is the coordinate matrix(for a robot with *n*-degrees of freedom) and $u_i \in \Re^n$ denotes the control inputs. However, one of the main shortcomings of this kinematic model is that it does not correspond to the dynamics of realistic agents. Therefore, in *Subsection 'B',* a general n-degrees of freedom dynamic model of real robots is considered to propose more realistic solutions for formation control of multi-agent systems.

A. Formation control of agents without considering mass of agents

To propose a control law, an artificial potential function is considered and pair wise potential fields are defined between agents as following

$$F_{ij} = L_{ij}\left(\left|z_i - z_j\right|\right), \forall i, j \in \{1,2,...,n\} \tag{2}$$

where L_{ij} is designed to define a proper inter-agent potential function. It is assumed that each agent senses the resultant potential of all other agents. The overall potential function is proposed to have the following form:

$$F = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} L_{ij}\left(\left|z_i - z_j\right|\right) + \sum_{i=1}^{N} Q_i\left(\left|z_i\right|\right) \tag{3}$$

This potential function is designed in order to have the following properties [11, 12]:

- Continuously differentiable
- Strictly convex
- Positive definite

For example, the following potential function can be chosen for a desired polygonal formation in 2D Space

$$F = \sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left(\left|z_i - z_j\right|^2 - d_{ij}\right)^2 + \sum_{i=1}^{N}\left(\left|z_i\right|^2 - r_i\right)^2 \tag{4}$$

To propose a solution for multi-agent formation control, the steepest descent direction is chosen [4, 11, 12], i.e.

$$f_i = \frac{\partial F}{\partial z_i}, \tag{5}$$

and the control law

$$u_i = -f_i, \forall i \in \{1,2,...,n\} \tag{6}$$

is proposed.

By substituting (6) in (1), the kinematic model is obtained as

$$\dot{z}_i = -f_i = -\frac{\partial F}{\partial z_i}, \forall i \in \{1,2,...,n\}, \tag{7}$$

which can be rewritten in the matrix form as following $\dot{Z} = -\nabla F$ where $Z = [z_1, z_2, ..., z_n]$ is the overall generalized coordinate vector.

In the next subsection, it is proposed to assume the multi-agent system with a general dynamic model. Furthermore, in Section 3 a sliding mode controller is used to force the satisfaction of (7). In other words, the proposed controller is designed to enforce the speed of each agent along the negative gradient of potential function defined in (3).

B. Formation control of robots with Dynamic models

Consider a group of *N* fully autonomous robots. The dynamics of the i^{th} simple robot is strongly nonlinear and can be written in the general form[17]:

$$M(z_i)\ddot{z}_i + C(z_i, \dot{z}_i)\dot{z}_i + g(z_i) = u_i \tag{8}$$

where $z_i \in \Re^n$ is the coordinate matrix(for a robot with *n*-degrees of freedom), $M(z_i) \in \Re^{n\times n}$ is a symmetric positive definite matrix and represents the inertia coefficients. $C(z_i, \dot{z}_i) \in \Re^{n\times n}$ is the matrix of centripetal, Coriolis, damping and rolling resistance forces, $g(z_i) \in \Re^n$ is an *n*-vector of gravitational forces and $u_i \in \Re^n$ denotes the control inputs.

3. CONTROLLER DESIGN METHODOLOGY

Sliding Mode Control (SMC) is an effective control methodology, which has been successfully applied to the field of robotic systems. Generally, SMC design consists of the following two main steps. First, the selection of a sliding surface which induces the stable reduced-order dynamics assigned by the designer. The second is designing a control law to force the closed-loop system trajectory onto the sliding surface (and keeping it on that surface)[17]. It should be mentioned that there is a little difference between traditional SMC and sliding model control design introduced here. In traditional SMC, the sliding surface is assumed constant. However, in multi-agent formation control sliding surface can vary when agents move.

In the proposed method of this paper, a sliding surface $S_i \in \Re^n$ representing the desired dynamics for the i^{th} robot is chosen as

$$S_i = \dot{z}_i + f_i \tag{9}$$

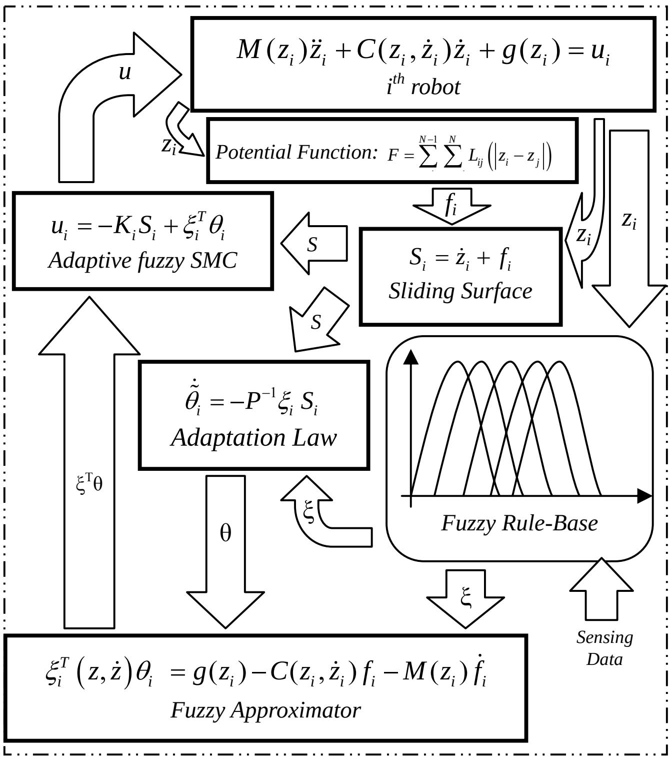


Figure 1- Block Diagram of Adaptive Fuzzy Sliding Mode Control Scheme

where z_i represents the coordinate vector of i^{th} robot (8) and f_i the gradient of potential function defined in (5). It is straightforward to write $\dot{S}_i$ as

$$\dot{S}_i = \ddot{z}_i + \dot{f}_i \tag{10}$$

Multiplying equation (10) by $M(z_i)$ and using some simple manipulations, lead to the following relation:

$$M(z_i)\dot{S}_i + C(z_i,\dot{z}_i)S_i = u_i - g(z_i) + C(z_i,\dot{z}_i) f_i + M(z_i)\dot{f}_i \tag{11}$$

Defining $A_i(z,\dot{z}) = g(z_i) - C(z_i,\dot{z}_i) f_i - M(z_i)\dot{f}_i$, equation (11) can be rewritten as

$$M(z_i)\dot{S}_i + C(z_i,\dot{z}_i)S_i = u_i - A_i(z,\dot{z}) \tag{12}$$

To design the reaching mode controller, following control law is proposed.

$$u_i = -K_i S_i + A_i(z,\dot{z}) \tag{13}$$

In order to use this control law, the function $A_i(.)$(i.e. $g(.)$, $C(.)$ and $M(.)$) must be known. However, in practice these functions may be unknown for most of real dynamical robots. To overcome this, we make use of an adaptive fuzzy logic system $\hat{A}_i()$ to approximate $A_i(.)$. Assuming the position and velocity of all robots is known, using the singleton fuzzifier, product inference, and weighted average defuzzifier[18], the output of the fuzzy model can be expressed as:

$$\hat{A}_i(z,\dot{z}) = \xi_i^T(z,\dot{z})\theta_i \tag{14}$$

where

$$\xi_i = \begin{bmatrix} \xi_{i1} & 0 \\ 0 & \xi_{i2} \end{bmatrix}, \theta_i = \begin{bmatrix} \theta_{i1} \\ \theta_{i2} \end{bmatrix}$$

Equation (14) suggests us to rewrite overall control law (13) as

$$u_i = -K_i S_i + \xi_i^T \theta_i \tag{15}$$

A Block diagram of the proposed control scheme is shown in Fig. 1.

It can be mentioned that, the issue of agent collision is not addressed directly in the proposed method. However, some small modifications on the artificial potential functions can handle this problem. The terms defined in (3) are known as attraction functions, and including inter-agent repulsion potentials as discussed in [4] can easily lead to the collision avoidance.

4. STABILITY ANALYSIS

To derive the adaptive law for adjusting θ_i, we first define the optimal parameter vector θ_i^* as

$$\theta_i^* = \arg\min_{\theta_i \in \Omega}\left[\sup\left\|\hat{A}_i(z,\dot{z}\,|\,\theta_i) - A_i(z,\dot{z})\right\|\right] \tag{16}$$

Define the minimum approximation error as

$$w_i = A_i(z,\dot{z}) - A_i(z,\dot{z}\,|\,\theta_i^*) \tag{17}$$

and it can be assumed that $w_i \in L_\infty$.

Based on(15), (16) and (17), Eq. (12) can be rewritten as

$$\begin{aligned} M(z_i)\dot{S}_i &= -K_i S_i - C(z_i,\dot{z}_i)S_i + \xi_i^T\theta_i - \xi_i^T\theta_i^* \\ &= -K_i S_i - C(z_i,\dot{z}_i)S_i + \xi_i^T\tilde{\theta}_i \end{aligned} \tag{18}$$

In the following theorem, the proposed control law (15) will be proved to be able of driving each individual robot (8) onto the sliding surface(9) (i.e. $S_i(t) = 0, \forall i \in \{1,2,\cdots,N\}$).

Theorem 1: Consider that uncertain nonlinear dynamic of the i^{th} robot is controlled by u_i in equation(15). Then each robot states trajectory converges to the sliding surface $s_i(t) = 0$ (9).

Proof: In order to derive the adaptive law for adjusting θ_i, we consider the Lyapunov candidate

$$V = \frac{1}{2}\sum_{i=1}^{N} S_i^T M S_i + \frac{1}{2}\sum_{i=1}^{N}\tilde{\theta}_i^T P_i \tilde{\theta}_i \ , \tag{19}$$

where $\tilde{\theta}_i = \theta_i - \theta_i^*$ and $P_i \in \Re^{n\times n}$ is an arbitrary positive definite matrix. Using(18), the time derivative of equation (19) is

$$\dot V = \frac{1}{2}\sum_{i=1}^{N}\left(\dot S_i^T M S_i + S_i^T M \dot S_i\right) + \frac{1}{2}\sum_{i=1}^{N}\left(\dot{\tilde\theta}_i^T P_i \tilde\theta_i + \tilde\theta_i^T P_i \dot{\tilde\theta}_i\right)$$
$$= \sum_{i=1}^{N} S_i^T M \dot S_i + \sum_{i=1}^{N} \tilde\theta_i^T P_i \dot{\tilde\theta}$$
$$= -\sum_{i=1}^{N}\left(S_i^T K_i S_i - S_i^T C(z_i,\dot z_i) S_i + S_i^T \xi_i^T \tilde\theta_i + S_i^T W_i\right) + \sum_{i=1}^{N} \tilde\theta_i^T P_i \dot{\tilde\theta}_i \quad (20)$$
$$\le \sum_{i=1}^{N}\left(-S_i^T K_i S_i + S_i^T \Gamma S_i + S_i^T W\right) + \sum_{i=1}^{N}\left(\tilde\theta_i^T \xi_i S_i + \tilde\theta_i^T P_i \dot{\tilde\theta}\right)$$

where Γ is defined as the upper bound of $C(z,\dot z)$:

$$C(z,\dot z) \le \Gamma \quad (21)$$

Above equation leads to the adaptive law as

$$\dot{\tilde\theta}_i = -P^{-1}\xi_i S_i \ . \quad (22)$$

Therefore we obtain

$$\dot V \le -\sum_{i=1}^{N} S_i^T (K_i - \Gamma) S_i + \sum_{i=1}^{N} S_i^T W_i \ , \quad (23)$$

and by selecting $K_i \ge \Gamma \Rightarrow K_i - \Gamma = K_i' \ge 0$, we have

$$\dot V \le -\sum_{i=1}^{N} S_i^T (K_i') S_i + \sum_{i=1}^{N} S_i^T W_i \ . \quad (24)$$

Based on universal approximation theorem[18], it is expected that the term $S_i^T W_i$ will be very small in the adaptive fuzzy approximator. Therefore, we have

$$\dot V \le 0 \ . \quad (25)$$

Using Barbalet's lemma[19], it can be shown reaching condition is satisfied. It implies $S_i(t) \to 0$ as $t \to 0$. So as mentioned in Theorem 1, $S_i(t) \to 0$ as $t \to 0$. ■

5. SIMULATION RESULTS

This section presents three simulation examples to illustrate the effectiveness of the proposed control scheme. In the first example, we give the simulation results of a simple formation control strategy for a group of six point mass-less agents with the simple kinematic model as (1). In the second, the performance of the SMC design is investigated in the case of six agents with known dynamics. Finally, the third example proves effectiveness of the proposed adaptive fuzzy SMC in case of formation control of six agents with fully unknown dynamics.

The unique hexagonal formation problem used in all three simulation examples, is defined by

$$F = \sum_{i=1}^{5}\sum_{j=i+1}^{6}\left(\left|z_i - z_j\right|^2 - d_{ij}\right)^2 + \sum_{i=1}^{6}\left(\left|z_i\right|^2 - r_i\right)^2 \ , \quad (26)$$

where $r_i = 1$ is the formation radius and d_{ij} is specified in Table 1.

Table 1- Parameter specifications of hexagonal formation

	|i-j|=1	|i-j|=2	|i-j|=3	|i-j|=4	|i-j|=5
d_{ij}	1.0	1.7	2.0	1.7	1.0

A. Point Mass-less Kinematic agent

Consider six agents with the simplest kinematic model $\dot z_i = -f_i$ and the hexagonal formation defined in (26). The initial positions of each robot is specified in following table (Table 2).

Table 2-Agents' initial positions

Agent No:	1	2	3	4	5	6
x_o	-5.0	-3.0	-1.0	+1.0	+3.0	+5.0
y_0	+1.0	-1.0	+1.0	-1.0	+1.0	-1.0

In this subsection, we use the simple control law defined in (5). Agent's motion trajectories are shown in Fig. 2.

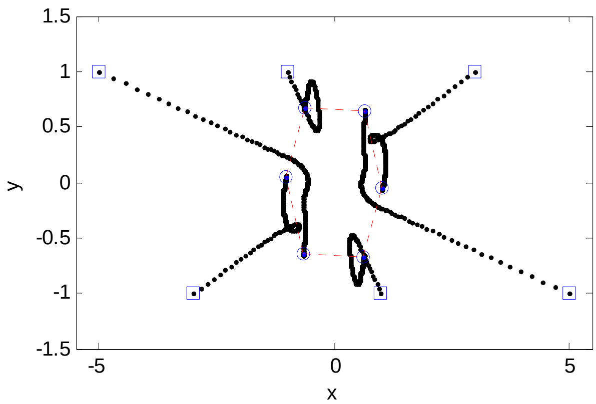


Figure 2- Hexagonal formation of point mass agents. Square marks show the initial positions of agents and Circle marks show the final positions.

In this simulation example, the effectiveness of the steepest descent based approach introduced in Subsection 2.A is proved to be effective.

B. Point Mass agents with known dynamic model

Consider a group of six mobile robots with known dynamic models. The dynamic of the i^{th} robot is considered as

$$\begin{bmatrix}0.6 & 0\\ 0 & 0.6\end{bmatrix}\begin{bmatrix}\ddot x_i\\ \ddot y_i\end{bmatrix} + \begin{bmatrix}0.15 & 0\\ 0 & 0.15\end{bmatrix}\begin{bmatrix}\dot x_i\\ \dot y_i\end{bmatrix} + \begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}\begin{bmatrix}\mathrm{sgn}(\dot x_i)\\ \mathrm{sgn}(\dot y_i)\end{bmatrix} = u_i \quad (27)$$

To give a solution for the formation problem (26), sliding surface is defined using (9) and the designed control law using (13) is proposed as following

$$\begin{bmatrix}u_x\\ u_y\end{bmatrix} = -\begin{bmatrix}0.15 & 0\\ 0 & 0.15\end{bmatrix}\begin{bmatrix}\dot x_i\\ \dot y_i\end{bmatrix} - \begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}\begin{bmatrix}\mathrm{sgn}(\dot x_i)\\ \mathrm{sgn}(\dot y_i)\end{bmatrix} - KS_i \quad (28)$$

The motion trajectories of this multi-agent system with the above parameters and the switching gain specified as

$$K = \begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}$$

is shown in Fig. 3.

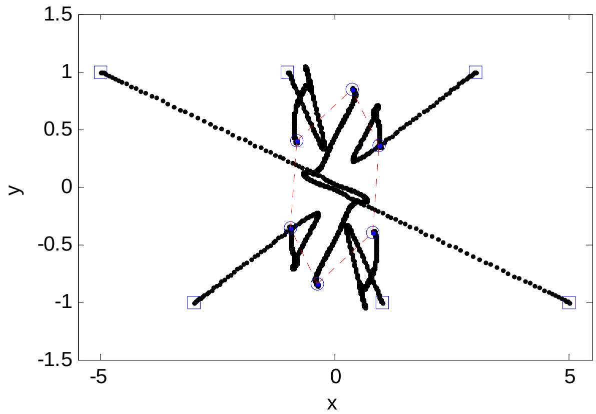


Figure 3-Hexagonal formation of agents with known dynamic model. Square marks show the initial positions of agents and Circle marks show the final positions.

C. Point Mass agent with unknown dynamics

To verify the effectiveness of proposed method, we present simulation results for a group of six agents with the same dynamic models as (27).

The formation problem and sliding surface are chosen as (26) and (9) respectively. However to design the control law, the dynamic model of agents' is assumed to be fully unknown. Therefore, a fuzzy logic approximator is used. Gaussian membership functions are defined and initial values of θ_i are chosen randomly. The learning rate in (22) is set to

$$P^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

Simulation results of the proposed adaptive fuzzy SMC scheme with agents' initial positions as Table 2 is shown in Fig.4.

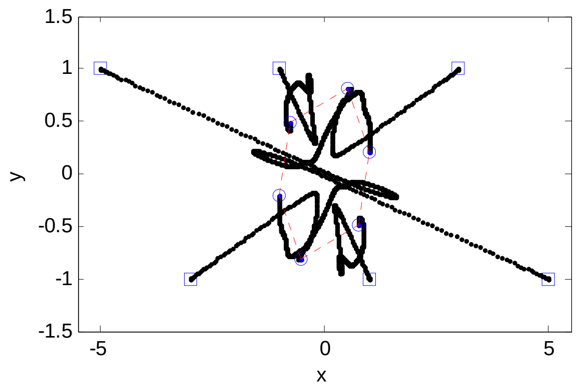


Figure 4-Hexagonal formation of agents with unknown dynamic model. Square marks show the initial positions of agents and Circle marks show the final positions

Effectiveness of the proposed scheme as a solution for formation control of multi-agent systems is shown by three simulation examples of this section. In the next section, a real test-bed is set up and a real experiment is presented using swarm of six real mobile robots.

6. EXPERIMENTAL VERIFICATION

In this section to prove the efficiency of proposed method, a real test-bed consisting six real mobile robots is set up. Detailed specifications of a single mobile robot (Fig. 5) is given in Table 3.

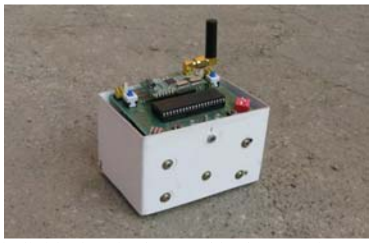


Figure 5-An individual real robot used in the experimental verification

Table 3-Specification of a real mobile robot used in experimental verification

Specification	***Value***	***Specification***	***Value***
Mass	580gr	*Length*	10cm
Max. Velocity	18cm/s	*Width*	6.5cm
Max. Acceleration	250m/s^2	*Height*	7cm

The swarm of six robots moves on a black rigid surface 210^{cm}x270^{cm}. *A* digital camera is installed at the height of 2.5m from the surface and an image-processing software uses 15 frames/second of an image with 600x800 pixels to exactly denote the place of each robot. Then the required raw data is sent to the robots using a wireless FSK module. Finally, each robot uses the received data to choose the proper velocity and direction of motion.

The physical model of a single robot is shown in Fig. 6.

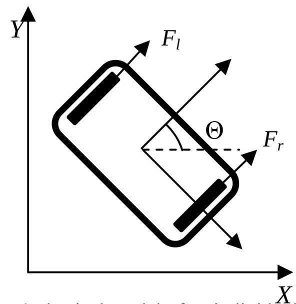


Figure 6-Physical model of an individual robot

Based on the physical model of the mobile robot, its constrained dynamic model is achieved as:

$$\begin{cases} m\ddot{x} = \left(f_r + f_l\right)\cos\theta \\ m\ddot{y} = \left(f_r + f_l\right)\sin\theta \\ I\ddot{\theta} - \lambda\left(\dot{y}.\sin\theta + \dot{x}.\cos\theta\right) = \dfrac{\left(f_r - f_l\right)d}{2} \\ \dot{y}\cos\theta\text{-}\dot{x}\sin\theta\text{=}0 \end{cases} \tag{29}$$

where *m* is the total mass of each robot, *I* is the central mass of inertia and *d* is the distance between two wheels of robot. As can be seen it is hard to find the close form of (29) in the normal form of (8). This would be more difficult to find the normal dynamic model of a constrained spatial mobile robot in a 3D space. Therefore, the proposed method in this research is the best solution for such formation control of mobile robots with unknown dynamics.

Experimental results of a real formation control for six mobile robots using the same control scheme as *Subsection '5.C'* is shown in Fig. 7. The agents' initial positions are chosen randomly given in Table 4 and the formation specification is the same as (26), however all its parameters are scaled down to half of their previous values.

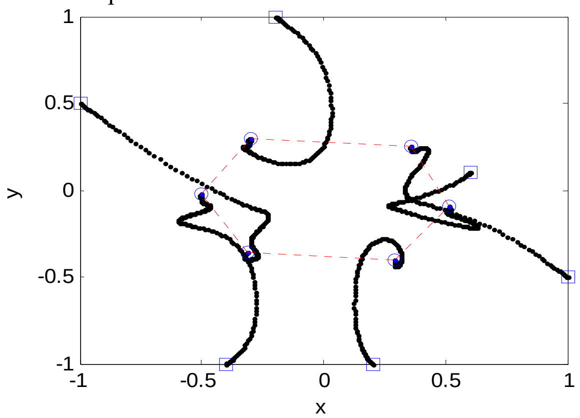
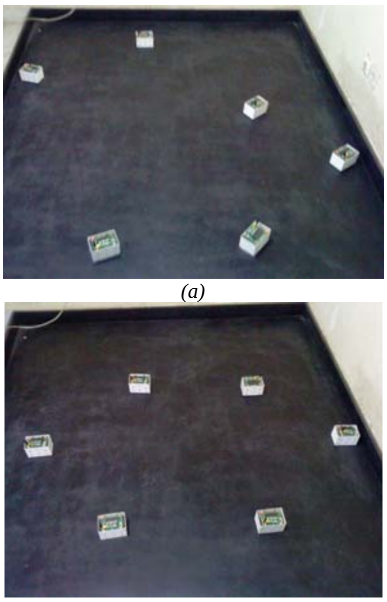


Figure 7-Hexagonal formation control of a real swarm of robots

Table 4-Agents' initial positions

Agent No:	*1*	*2*	*3*	*4*	*5*	*6*
x_o	-1.0	-0.4	-0.2	+0.2	+0.6	+1.0
y_0	+0.5	-1.0	+1.0	-1.0	+0.1	-0.5

A real picture of the final formation of swarm robots is presented in Fig. 8.



(a)

(b)

Figure 8-Hexagonal formation control of a real group of robots (a)Initial positions (b)Final positions

7. CONCLUSIONS

In this paper, the formation control problem of a class of multi-agent systems with fully unknown dynamics was investigated. On the basis of the Lyapunov stability theory, a new adaptive controller with corresponding parameter update laws was developed. All the theoretical results were verified by simulation examples to demonstrate the effectiveness of the proposed control scheme. Finally, a swarm of six real mobile robots is used to prove the applicability of control scheme in real applications.

REFERENCES

[1] H. Berghuis and H. Nijmeijer, *A passivity approach to controller-observer design for robots.* Robotics and Automation, IEEE Transactions on, 1993. **9**(6): p. 740-754.

[2] W. Oelen, H. Berghuis, H. Nijmeijer, and C. Canudas de Wit, *Hybrid stabilizing control on a real mobile robot.* Robotics & Automation Magazine, IEEE, 1995. **2**(2): p. 16-23.

[3] O.J. Sordalen and C. Canudas de Wit, *Exponential control law for a mobile robot: extension to path following.* Robotics and Automation, IEEE Transactions on, 1993. **9**(6): p. 837-842.

[4] J.H. Reif and H. Wang, *Social potential fields: A distributed behavioral control for autonomous robots.* Robotics and Autonomous Systems, 1999. **27**(3): p. 171-194.

[5] K. Watanabe and A. Nakamura. *A design of tiny basin test-bed for AUV multi agent*. in *OCEANS, 2005. Proceedings of MTS/IEEE*. 2005.

[6] B.A. White, A. Tsourdos, I. Ashokaraj, S. Subchan, and R. Zbikowski, *Contaminant Cloud Boundary Monitoring Using Network of UAV Sensors.* Sensors Journal, IEEE, 2008. **8**(10): p. 1681-1692.

[7] R. Sepulchre, D.A. Paley, and N.E. Leonard, *Stabilization of Planar Collective Motion: All-to-All Communication.* Automatic Control, IEEE Transactions on, 2007. **52**(5): p. 811-824.

[8] D.V. Dimarogonas and K.J. Kyriakopoulos, *Connectedness Preserving Distributed Swarm Aggregation for Multiple Kinematic Robots.* Robotics, IEEE Transactions on, 2008. **24**(5): p. 1213-1223.

[9] M. Proetzsch, T. Luksch, and K. Berns, *Development of complex robotic systems using the behavior-based control architecture iB2C.* Robotics and Autonomous Systems, 2010. **58**(1): p. 46-67.

[10] K. Peng and Y. Yang, *Leader-following consensus problem with a varying-velocity leader and time-varying delays.* Physica A: Statistical Mechanics and its Applications, 2009. **388**(2-3): p. 193-208.

[11] C.C. Cheaha, S.P. Houa, and J.J.E. Slotine, *Region-based shape control for a swarm of robots.* Automatica, 2009. **45**(10): p. 2406-2411.

[12] V. Gazi, *Swarm aggregations using artificial potentials and sliding-mode control.* Robotics, IEEE Transactions on, 2005. **21**(6): p. 1208-1214.

[13] L.A. Zadeh, *Fuzzy Sets.* Information and Control, 1965. **8**(3): p. 338-353.

[14] N. Noroozi, M. Roopaei, P. Karimaghaee, and A.A. Safavi, *Simple adaptive variable structure control for unknown chaotic systems.* Communications in Nonlinear Science and Numerical Simulation, 2010. **15**(3): p. 707-727.

[15] P. Shahmaleki, M. Mahzoon, and B. Ranjbar. *Real time experimental study of truck backer upper problem with fuzzy controller*. in *World Automation Congress (WAC)*. 2008.

[16] T. Das and I.N. Kar, *Design and implementation of an adaptive fuzzy logic-based controller for wheeled mobile robots.* Control Systems Technology, IEEE Transactions on, 2006. **14**(3): p. 501-510.

[17] E. Slotine and W. Li, *Applied Nonlinear Control ;Prentice-Hall, Englewood Cliffs, NJ.* 1991.

[18] L.X. Wang, *A Course in Fuzzy Systems and Control*. 1997, Englewood Cliffs, NJ: Prentice-Hall.

[19] H.K. Khalil, *Nonlinear systems (2nd ed.)*. 1996, Englewood Cliffs, NJ: Prentice Hall.

