Theory of Cooperation in Complex Social Networks (Supplemental Material)

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Abstract

For completeness, this appendix provides additional experimental results conducted on various network types and sizes to further validate the theorems presented in the original paper[1]. Different statistical criteria are adopted to pave the way for exploring the potential correlation between network topologies, initial configurations and final agreements.

Experimental Validation

One of the goals of this work is to explore the dependencies between *cooperation promotion* and the initial configuration of social networks. Theorems 1 and 4 of the original paper prove that there exists a direct relation between the trivial left eigenvector of the model's state-transition matrix and the amount of contribution each individual exhibits in the final agreement. Furthermore, Theorems 2, 3, and 5 prove that the adaptation rate of one or a group of individuals plays a prominent role in determining the final agreement. Based on these results, the original paper proposes different mechanisms in which the final agreement of a network can be influenced by adopting multi-rate action matrices.

In this appendix, we amend the results of the original paper by presenting an in-depth empirical analysis of different network types and sizes. On a high level, two experiments are performed. Firstly, results on *evolutionary* networks with varying sizes and degree distributions are reported. Both final agreement conclusions as well as control mechanism results are confirmed. Secondly, these are repeated for the *coevolutionary* networks case.

This study is conducted on 12 different network types, from which 6 are scale free and 6 are small-world. Different network sizes varying from 50 to 1000 individuals have been considered. Network specific parameters have also been varied to cover a broader range of applications. Namely, scale-free networks with connectivities of 2 and 4, and small-world networks with 0.1 and 0.4 rewiring probabilities have been studied.

[1]B. Ranjbar-Sahraei, H. Bou Ammar, D. Bloembergen, K. Tuyls, and G. Weiss, "Theory of Cooperation in Complex Social Networks", in proceedings of *28th AAAI Conference on Artificial Intelligence*, 2014.

Four measures are used to report the results. Namely, $\min$, $\max$, mean and std (i.e., standard deviation) are adopted to reflect the overall behavior in these different networks. Furthermore, instead of node degree distributions, which ignore the states of individuals, cooperator and defector fitnesses (F_c and F_d, respectively) were considered. Table 1 represents these configurations for the 12 different networks used.

Four different experiments are implemented. First, the simple evolution of behaviors in which the fitnesses remain fixed and behaviors evolve through the time. Second, the same evolutionary network where all the cooperators' update rate is decreased from 1.0 to 0.1; in this experiment we expect an increase in the final agreement. The coevolutionary behavior of the same networks are studied next, and finally the coevolution in presence of a decrement of cooperators' update rate from 1.0 to 0.1 is included.

Evolutionary Network Experiments

Two scenarios are considered for evolutionary networks. Firstly, behavioral evolution, with fixed fitnesses, in which convergence to a final agreement is studied. Results shown in Table 2 (i.e., first 9 rows) demonstrate that a final agreement can be reached for both the scale free and small-world networks in accordance to the conclusions provided in the original paper.

In the second set of experiments, multi-rate state update has been tested. Here, the same evolutionary networks are used. The cooperators' update rate, however, is decreased from 1.0 to 0.1. According to the results presented in the original paper, individuals with low update rate contribute most to the final agreement. Therefore, in this case an increase in the final agreement has to be observed compared to the previous experiments. Results shown by last 9 rows of Table 2, clearly demonstrate this increase.

Coevolutionary Network Experiments

The same experiments are then repeated on different types of coevolutionary networks. Results shown in Table 3 again confirm the conclusions attained in the original paper. It is worth noting that although no closed form solution can be attained in multi-rate state update scenario for coevolutionary networks, results again demonstrate that as the update rate of an individual decreases, its contribution to the final agreement increases.

Table 1: Configuration used for each of the 12 networks showing the minimum, maximum, mean and standard deviation for each of the defectors and cooperators fitnesses as a function of the size, connectivity and wiring probability for the two network types used.

	Scale Free Networks						Small World Networks					
	N = 50 C = 2	N = 50 C = 4	N = 500 C = 2	N = 500 C = 4	N = 1000 C = 2	N = 1000 C = 4	N = 50 P_W = 0.1	N = 50 P_W = 0.4	N = 500 P_W = 0.1	N = 500 P_W = 0.4	N = 1000 P_W = 0.1	N = 1000 P_W = 0.4
min(F_d)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
max(F_d)	12.000	24.000	92.000	104.000	64.000	204.000	12.000	20.000	20.000	20.000	20.000	24.000
mean(F_d)	4.160	7.040	3.616	7.936	3.976	8.160	6.720	9.760	7.648	7.632	8.176	7.992
std(F_d)	2.939	6.141	7.886	10.253	6.374	13.475	3.208	5.044	4.025	4.263	4.142	4.324
min(F_c)	-1.000	-2.000	-3.000	-5.000	-6.000	-6.000	-4.000	-4.000	-4.000	-7.000	-5.000	-6.000
max(F_c)	17.000	19.000	12.000	59.000	34.000	40.000	12.000	11.000	15.000	18.000	15.000	17.000
mean(F_c)	3.280	4.480	1.592	3.992	2.114	3.492	5.280	2.600	4.304	4.548	3.764	3.966
std(F_c)	3.635	5.316	2.881	6.500	3.728	5.294	5.136	3.428	4.073	4.476	4.019	3.979

Table 2: Final agreement and multi-rate update results on 12 different evolutionary networks. Each column corresponds to a network type with specific connectivity/wiring probability and number of individuals. r_c and r_d denote the weights corresponding to cooperators and defectors; respectively. Results clearly confirm the conclusions attained in the original paper.

		Scale Free Networks						Small World Networks					
		N = 50 C = 2	N = 50 C = 4	N = 500 C = 2	N = 500 C = 4	N = 1000 C = 2	N = 1000 C = 4	N = 50 P_W = 0.1	N = 50 P_W = 0.4	N = 500 P_W = 0.1	N = 500 P_W = 0.4	N = 1000 P_W = 0.1	N = 1000 P_W = 0.4
Evolutionary Network	**min(r_d)**	0.000	0.002	0.000	0.000	0.000	0.000	0.002	0.002	0.000	0.000	0.000	0.000
	max(r_d)	0.253	0.274	0.268	0.284	0.151	0.040	0.098	0.109	0.029	0.029	0.015	0.068
	std(r_d)	0.050	0.057	0.029	0.025	0.011	0.003	0.025	0.035	0.004	0.004	0.002	0.006
	sum(r_d)	0.321	0.704	0.928	0.876	0.801	0.714	0.475	0.894	0.730	0.658	0.787	0.866
	min(r_c)	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000
	max(r_c)	0.626	0.110	0.003	0.047	0.127	0.009	0.076	0.017	0.014	0.021	0.003	0.016
	std(r_c)	0.125	0.024	0.001	0.003	0.006	0.001	0.023	0.003	0.001	0.002	0.000	0.001
	sum(r_c)	**0.679**	**0.296**	**0.072**	**0.124**	**0.199**	**0.296**	**0.525**	**0.106**	**0.270**	**0.342**	**0.213**	**0.134**
	Final Agreement	**0.675**	**0.297**	**0.071**	**0.115**	**0.202**	**0.304**	**0.520**	**0.106**	**0.273**	**0.341**	**0.229**	**0.133**
Multirate Evolutionary Network	**min(r_d)**	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000
	max(r_d)	0.068	0.126	0.208	0.190	0.084	0.019	0.032	0.076	0.014	0.012	0.008	0.044
	std(r_d)	0.013	0.026	0.023	0.017	0.006	0.002	0.008	0.025	0.002	0.002	0.001	0.004
	sum(r_d)	0.086	0.322	0.720	0.586	0.446	0.333	0.153	0.628	0.351	0.278	0.426	0.563
	min(r_c)	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000
	max(r_c)	0.843	0.251	0.013	0.157	0.353	0.021	0.123	0.060	0.034	0.045	0.009	0.052
	std(r_c)	0.168	0.054	0.002	0.010	0.016	0.002	0.036	0.010	0.003	0.005	0.001	0.003
	sum(r_c)	**0.914**	**0.678**	**0.280**	**0.414**	**0.554**	**0.667**	**0.847**	**0.372**	**0.619**	**0.722**	**0.574**	**0.437**
	Final Agreement	**0.912**	**0.678**	**0.278**	**0.409**	**0.547**	**0.660**	**0.835**	**0.372**	**0.614**	**0.744**	**0.541**	**0.455**

Table 3: Final agreement and multi-rate update results on 12 different coevolutionary networks. Each column corresponds to a network type with specific connectivity/wiring probability and number of individuals. v_c and v_d denote the weights corresponding to cooperators and defectors; respectively. Results clearly confirm the conclusions attained in the original paper.

		Scale Free Networks						Small World Networks					
		N = 50 C = 2	N = 50 C = 4	N = 500 C = 2	N = 500 C = 4	N = 1000 C = 2	N = 1000 C = 4	N = 50 P_W = 0.1	N = 50 P_W = 0.4	N = 500 P_W = 0.1	N = 500 P_W = 0.4	N = 1000 P_W = 0.1	N = 1000 P_W = 0.4
Coevolutionary Network	**min(v_d)**	0.000	0.004	0.000	0.000	0.000	0.000	0.014	0.014	0.001	0.001	0.001	0.000
	max(v_d)	0.014	0.117	0.408	0.062	0.005	0.053	0.028	0.047	0.006	0.006	0.004	0.004
	std(v_d)	0.003	0.025	0.026	0.005	0.001	0.003	0.004	0.009	0.001	0.001	0.000	0.000
	sum(v_d)	0.089	0.491	0.638	0.507	0.069	0.577	0.543	0.679	0.587	0.570	0.609	0.598
	min(v_c)	0.000	0.003	0.000	0.000	0.000	0.000	0.008	0.007	0.001	0.000	0.000	0.000
	max(v_c)	0.837	0.161	0.171	0.080	0.857	0.026	0.028	0.031	0.004	0.008	0.002	0.004
	std(v_c)	0.167	0.041	0.011	0.006	0.038	0.002	0.006	0.005	0.000	0.001	0.000	0.000
	sum(v_c)	**0.911**	**0.509**	**0.362**	**0.493**	**0.931**	**0.423**	**0.457**	**0.321**	**0.413**	**0.430**	**0.391**	**0.402**
	Final Agreement	**0.909**	**0.495**	**0.353**	**0.490**	**0.920**	**0.420**	**0.455**	**0.316**	**0.410**	**0.427**	**0.388**	**0.401**
Multirate Coevolutionary Network	**min(v_d)**	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.004	0.000	0.000	0.000	0.000
	max(v_d)	0.003	0.048	0.276	0.025	0.002	0.021	0.015	0.039	0.004	0.006	0.003	0.004
	std(v_d)	0.001	0.011	0.019	0.002	0.000	0.001	0.003	0.009	0.001	0.001	0.000	0.000
	sum(v_d)	0.019	0.167	0.470	0.176	0.019	0.234	0.216	0.355	0.246	0.238	0.272	0.268
	min(v_c)	0.000	0.002	0.000	0.000	0.000	0.000	0.015	0.011	0.001	0.000	0.000	0.000
	max(v_c)	0.903	0.412	0.113	0.154	0.901	0.047	0.042	0.052	0.009	0.027	0.004	0.052
	std(v_c)	0.180	0.087	0.009	0.012	0.040	0.003	0.008	0.008	0.001	0.003	0.000	0.002
	sum(v_c)	**0.981**	**0.833**	**0.530**	**0.824**	**0.981**	**0.766**	**0.784**	**0.645**	**0.754**	**0.762**	**0.728**	**0.732**
	Final Agreement	**0.979**	**0.835**	**0.534**	**0.814**	**0.979**	**0.755**	**0.771**	**0.639**	**0.758**	**0.754**	**0.703**	**0.727**